Acta Technica Napocensis: Civil Engineering & Architecture Vol. 55, No. 3 (2012)
***Journal homepage:* http://constructii.utcluj.ro/ActaCivilEng**

Global structural analysis of central cores supported tall buildings compared with FEM

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Received 15 May2012; Accepted 10 November 2012

Abstract

The focus of this article is to present an approximate method of calculation based on the equivalent column theory. This approximate method of calculation may be successfully applied in the case of tall buildings. Knowing the geometrical and stiffness characteristics of the structure, applying the equivalent column theory may determined: the displacements in both directions, the rotation of the structure, critical load, shear forces, bending moments for each resisting element and the torsional moment of the structure. The results obtained using the approximate calculation method will be compared with the results obtained using an exact calculation based on F.E.M.: Autodesk Robot Structural Analysis and ANSYS 12.1.

Rezumat

In acest articol se doreste prezentarea unei metode de calcul aproximative bazate pe teoria stalpului echivalent. Metoda de calcul aproximativa poate fi aplicata cu succes si in cazul cladirilor inalte. Cunoscand carcateristicile geometrice si de rigiditate ale structurii, cu ajutorul teoriei stalpului echivalent se pot calcula: deplasarile pe cele doua directii, rotirea structurii, incarcarea critica, forta taietoare, momentul incovoietor pentru fiecare element de rezistenta, momentul de torsiune al structurii. Rezultatele obtinute prin metoda de calcul aproximativa vor fi comparata cu rezultatele obtinute utilizand o metoda de calcul exacta bazata pe M.E.F.: Autodesk Robot Structural Analysis si ANSYS 12.1.

Keywords: tall building, central core, equivalent column theory, FEM

1. Introduction

The aim of this article is to achieve an approximate analysis of multi-levels structures under horizontal loads. The shear walls and central cores ensure the lateral stiffness of the structure and

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resist the horizontal loads. This structural analysis is based on the equivalent column's theory that can be applied for multi-levels structures. The design of a tall building is problematic both architectural and structural. From a structural point of view, the main problems that appear to multi-levels buildings are related to the effects of horizontal loads and how the relative displacements can be limit (Taranath Bungale).

Another problem that occurs to high-rise structures is represented by buildings vibrations, this problem will also be treated in this approximate calculation method. With the increase of building's height, the classical methods of structural analysis can not be applied, thus will apply a global analysis and the whole structure will be considered as a single column cantilever.

Once with the appearance of powerful computers and software based on finite element method, can achieve a three-dimensional analysis of structural models with large number of bays and levels. The structural analysis based on F.E.M. has a high level of accuracy and structural detailing, thus this calculation method can be considered as an exact method. Nevertheless there are numerous authors which present the structural disadvantages of these models. FEM programs provide a quick result for a particular building, but cannot answer the general question how the building response is governed by decisive structural parameters (Steenbergen si Blaauwendraad, 2007). Although computer programs based on FEM are well developed, errors can occur because of a large number of data entering the calculation or due to results interpretation. To avoid obtaining incorrect results, can choose to achieve a comparative study of the structural analysis based on the FEM, and another approximate structural analysis, based on the equivalent column's theory.

Thus, an alternative for structural analysis based on FEM is represented by an approximate calculation, based on a global structural analysis of tall buildings. The global analysis takes into account only the predominant characteristics of the building.

At the same time, this approximate structural analysis is a fast calculation method which provides results close to those obtained using the exact calculation. Thus, for a first stage of design analysis, when structural concept is not established exactly, the global analysis represents a fast and efficient method of calculation.

In this article is performed a comparative study between the results obtained by the approximate method of calculation based on equivalent column theory and the exact method based on FEM, for central core structure.

It is important to know the theories behind structural analysis, not only to verify and compare the results obtained by FEM but also to develop new structural computer programs.

2. Equivalent column's theory

The global structural analysis of reinforced concrete tall buildings is based on the equivalent column's theory, respecting the civil engineering theorems. The approximate calculation method analyzes lateral loads distribution to shear walls and central cores structural systems. To simplify the structural model used in the design software, will consider only those elements able to resist lateral loads (wind and earthquake).

There are several authors who presented and developed approximate computing methods to determine the distribution of lateral loads in tall buildings: Bungale Taranath, Brian Smith, Karoly Zalka. Equivalent column theory can be applied for regular structures, where the geometric and stiffness characteristics of structural elements are constant throughout the building's height [1]:
(a) the material of the structures is homogeneous, isotropic and obeys Hooke's law;
(b) the floor slabs are stiff in their plane and flexible perpendicularly to their plane;
(c) the structures have no geometrical imperfections, they develop small deformations and the third-order effect of the axial forces is negligible;
(d) the loads are applied statically and maintain their direction (they are conservative forces);
(e) the location of the shear center only depends on geometrical characteristics;

Central cores are considered space systems capable of resisting lateral loads is both directions. In both models of analysis applied to multi-levels structure will take into account the spatial behavior of central cores. The main advantage of spatial structures is the ability to resist shear forces, bending moments in both directions as well as torque, since the torsional stiffness of central cores is large. The central core behavior in bending and torsion is similar to that of a thin-walled bar (Vlasov). The structural deformation is influenced also by the rotation of foundation, but this aspect is neglected by considering the equivalent column fixed at the base.

The structural elements able to resist lateral loads, shear walls and central cores in this case, will be reduced to an equivalent cantilever column, whose bending and torsional stiffness represents the whole structure's stiffness. Using column analogy theory, the whole structure will become a static determined structure. The equivalent column is situated in the shear central of the structure; depending on the geometrical and stiffness characteristics of the structural elements.
The shear central position is given by [1]:

$$\overline{x_0} = \frac{I_{xy}\left(\sum_1^n I_{y,i}\overline{y_i} - \sum_1^n I_{xy,i}\overline{x_i}\right) - I_y\left(\sum_1^n I_{xy,i}\overline{y_i} - \sum_1^n I_{x,i}\overline{x_i}\right)}{I_x I_y - I_{xy}^2}$$

$$\overline{y_0} = \frac{I_x\left(\sum_1^n I_{y,i}\overline{y_i} - \sum_1^n I_{xy,i}\overline{x_i}\right) - I_{xy}\left(\sum_1^n I_{xy,i}\overline{y_i} - \sum_1^n I_{x,i}\overline{x_i}\right)}{I_x I_y - I_{xy}^2}$$

Where: $I_{x,i}$, $I_{y,i}$ – the moments of inertia for both principal directions, $I_{xy,i}$ – the product of inertia; x_i, y_i – the distances from the shear center to the centroid center of every element, I_w – warping constant, and also J – Saint-Venant torsional constant.
It is also necessary to determine the moment of inertia I_X, the product of inertia I_{XY}, the warping constant I_W and the Saint-Venant torsional constant J for the whole structural system reduced to an equivalent column, by using the following relationships [1][2]:

$$I_X = \sum_1^n I_{x,i} \quad ; \quad I_Y = \sum_1^n I_{y,i} \quad ; \quad I_{XY} = \sum_1^n I_{xy,i} \quad ; \quad J = \sum_1^n J_i \tag{1}$$

$$I_w = \sum_1^n I_{w,i} + I_{x,i}x_i^2 + I_{y,i}y_i^2 - 2I_{xy,i}x_i y_i \tag{2}$$

The above relations represent the building's characteristics required for determining the global behavior of the structure. The first 3 characteristics are important for the global bending behavior and the last 2 characteristics J and Iw represent the torsional characteristics of the equivalent column.

The radius of gyration is a structural characteristic required for stability analysis and it is determined according to the structural loads and the in plan area of the building.

$$i_p = \sqrt{\frac{\int q(x,y)(x^2+y^2)dA}{\int q(x,y)dA}} \tag{3}$$

If the plan's structure is rectangular, the formula for determining the radius of gyration is simplified and depends only on the size of the building's in plan *L*, *B* and *t* – the distance between the shear central and the centroid of the building.

$$i_p = \sqrt{\frac{L^2+B^2}{12}+t^2} \tag{4}$$

Based on the equivalent column theory was developed a calculation soft-ware using Matlab, which determine: critical load, structural frequency, maximum displacements in both directions, rotation, shear forces and bending moments of the structural elements and torsional moments: M_t – Saint-Venant torsional moment si M_ω- warping torsion moment.
The inputs data of the computer program are all the geometrical and stiffness characteristics of the structural elements, upon which will determine the equivalent column characteristics.

2.1. Critical load

The critical loads in *x* and *y* direction, in case of equivalent column theory, are based on the Timoshenko relations for a cantilever column loaded uniformly distributed.

$$\mathrm{N_{cr,X}} = \frac{7{,}84\ \mathrm{r_s EI_Y}}{\mathrm{H}^2} \quad ; \quad \mathrm{N_{cr,Y}} = \frac{7{,}84\ \mathrm{r_s EI_X}}{\mathrm{H}^2} \tag{5}$$

The difference between Timoshenko formulas and the one presented above by K. Zalka is the reduction factor $r_s = n/(n+1.60)$, which takes into account that the vertical loads are concentrated loads level and not uniformly distributed load over the building's height.
The critical load for pure torsion [1]:

$$\mathrm{N_{cr,\varphi}} = \frac{\alpha \mathrm{r_s EI_\omega}}{\mathrm{i_p^2 H^2}} \tag{6}$$

$$k_s = \frac{k}{\sqrt{r_s}}; \quad k = H\sqrt{\frac{GJ}{EI_w}} \tag{7}$$

Where: n-number of levels

α– critical load parameter as a function of the parameter k_s

EI_ω- warping rigidity of the core

GJ – shear torsional rigidity

2.2. Fundamental frequency

The fundament frequency of the structure represents an essential characteristic for the dynamic analysis of the building. An approximate calculation for the building's frequency using Timoshenko's formula:

$$f_X = \frac{0{,}56 r_f}{H^2}\sqrt{\frac{EI_Y}{\varphi A}};\quad f_Y = \frac{0{,}56 r_f}{H^2}\sqrt{\frac{EI_X}{\varphi A}};\quad f_\varphi = \frac{\eta r_f}{i_p H^2}\sqrt{\frac{EI_\omega}{\varphi A}} \tag{8}$$

The main geometrical characteristic of the structure in determining the fundamental frequency is the building's height. As the building height is greater the fundamental frequency becomes lower. The design code ASCE7 makes a differentiation between the rigid and flexible buildings according to the building's frequency value. The rigid structures have a natural frequency equal or greater than 1Hz. The design code ASCE7 gives the formula to determine the natural frequency in case of a cantilever with constant section; without taking into account the reduction factor:

$$r_f = \sqrt{n/(n+2.06)} \tag{9}$$

The period of vibration in *x* and *y* direction is determined using the following relations:

$$T_X = \frac{1.787H^2}{r_f}\sqrt{\frac{\varphi A}{EI_Y}};\qquad T_Y = \frac{1.787H^2}{r_f}\sqrt{\frac{\varphi A}{EI_X}} \tag{10}$$

2.3. Maximum displacements

The whole structure is replaced by a cantilever column with a constant stiffness throughout the building's height. The governing differential equations defining the unsymmetrical bending and torsion of the equivalent column assume the following form in *x*-*y*-*z* coordinate system [Vlasov 1940].

$$EI_y\frac{d^4u}{dz^4} + EI_{xy}\frac{d^4v}{dz^4} = q_z(z)$$
$$EI_x\frac{d^4v}{dz^4} + EI_{xy}\frac{d^4u}{dz^4} = q_z(z)$$
$$EI_w\frac{d^4\varphi}{dz^4} - GJ\frac{d^4\varphi}{dz^4} = m_z(z) \tag{11}$$

The first two equations defines the equivalent column displacements in both directions *x* and *y*, while the third equation defines the equivalent column torsion. If the bracing elements are symmetrically arranged, the product of inertia is zero $I_{xy}=0$, and the first two equations given by Vlasov will be simplified and will remain only the first term on the left side of the equation.

The equivalent column is a vertical cantilever fixed at the base. Thus, using the boundary conditions will determine the equations of displacement in both directions and the equation of rotation [1].

- lateral displacements and rotation are zero at the fixed end:
  $$u(0) = v(0) = \varphi(0) = 0$$
- at the fixed bottom no warping develops:
  $$u'(0) = v'(0) = \varphi'(0) = 0$$
- at the top of the column the bending moments and warping stresses are zero
  $$u''(H) = v''(H) = \varphi''(H) = 0$$
- at the top of the column the shear forces and the torsional moments are zero
  $$u'''(H) = v'''(H) = EI_\omega\varphi'''(H) - GJ\varphi'(H) = 0$$

Integrating the equations given by Vlasov and taking into consideration the boundary condition mentioned above, will determine the general displacement equations and the maximum displacement of the equivalent column in both directions, using the equations [1]:

$$u_{max}(H) = \frac{\overline{q_x}}{E}\left(\frac{1}{8} + \mu\frac{11}{120}\right)H^4$$
$$v_{max}(H) = \frac{\overline{q_y}}{E}\left(\frac{1}{8} + \mu\frac{11}{120}\right)H^4 \qquad (12)$$

Where: $\overline{q_x} = \frac{I_x q_{0x} - I_{xy} q_{0y}}{I_x I_y - I_{xy}^2}$ $\quad \overline{q_y} = \frac{I_y q_{0y} - I_{xy} q_{0x}}{I_x I_y - I_{xy}^2}$

μ- the slope of the trapezoidal load

Similarly will determine the rotation of the equivalent column:

$$\varphi(z) = \frac{m_{zo}H^2}{k^2 GJ coshk}\left\{(1+\mu)\left(cosh\frac{kz}{H} - 1\right) + \left(1 + \frac{\mu}{2} - \frac{\mu}{k^2}\right)k\left(\sinh\left(k - \frac{kz}{H}\right) - sinhk\right) + \frac{zk^2}{H^2}coshk\left(H - \frac{z}{2} + \mu\left(\frac{H}{2} - \frac{H}{k^2} - \frac{z^2}{6H}\right)\right)\right\} \qquad (13)$$

The maximum rotation appears at the top of the equivalent column, thus to determine the maximum rotation of the building, *z* is equal to building's height. If $m_{zo} = 0$ can notice from the above relation that the equivalent column rotation is zero, in this case the horizontal load passes through the shear center of the structure. Thus, m_{zo} is determined using the relation:

$$m_{z0} = q_x y_C + q_y x_C$$

Where: (x_C, y_C) represents the centroid of the building.

2.4. Shear force

The equivalent column transmits the horizontal loads to bracing elements of the structure through the slabs considered as infinite rigid in their plane. The bracing elements of the structure resist shear forces; thus, will appear bending moments in resistance elements and by slabs rotation will appear torsion in elements.

The shear forces that appear in each structural elements are obtained be integrating the relation that defines the external loads; taking into account that shear forces at the top of the equivalent column are zero [1].

$$T_{x,i} = \left(H - z - \mu\left(\frac{z^2}{2H} - \frac{H}{2}\right)\right)\left(I_{y,i}\overline{q_x} + I_{xy,i}\overline{q_y}\right) - \frac{I_{y,i}y_i - I_{xy,i}x_i}{I_\omega}m_{z0}H\eta_T$$
$$Ty,i = \left(H - z - \mu\left(\frac{z^2}{2H} - \frac{H}{2}\right)\right)\left(I_{x,i}\overline{q_y} + I_{xy,i}\overline{q_x}\right) + \frac{I_{x,i}x_i - I_{xy,i}y_i}{I_\omega}m_{z0}H\eta_T \qquad (14)$$

It can be notice that the first part of the equation defines the shear center that appears due to lateral displacements and the second part due to structural system rotation.

Where: η_T – represents the shear force factor and is determined as a function of k parameter and μ- the slope of the trapezoidal load

$$\eta_T = \frac{1}{kcoshk}\left\{\left(1+\frac{\mu}{2}-\frac{\mu}{k^2}\right)\left(\cosh\left(k-\frac{kz}{H}\right)-1\right)k-(1+\mu)(sinh\frac{kz}{H}-sinhk)\right\} \qquad (15)$$

The parameter k is determined according to torsional stiffness and warping stiffness of the equivalent column. The value of parameter k increases with the torsional rigidity St. Venant.
The maximum shear forces in case of a vertical cantilever appear at the fixed end of the equivalent column. Thus, to determine the maximum shear forces for each element, are used the above relations for z=0.

2.5. Bending moment

The relations that determine bending moments are obtained by integrating the relations that define shear forces; taking into account that bending moment at the top of the column is zero.

$$\mathrm{M_{x,i}} = -\left(\frac{(\mathrm{z}-\mathrm{H})^2}{2}+\mu\left(\frac{\mathrm{z}^3}{6\mathrm{H}}-\frac{\mathrm{zH}}{2}+\frac{\mathrm{H}^2}{3}\right)\right)\left(\mathrm{I_{y,i}}\bar{\mathrm{q}}_\mathrm{x}+\mathrm{I_{xy,i}}\bar{\mathrm{q}}_\mathrm{y}\right)+\frac{\mathrm{I_{y,i}y_i}-\mathrm{I_{xy,i}x_i}}{\mathrm{I}_\omega}\frac{\mathrm{m_{z0}H^2}}{2}\eta_\mathrm{M}$$

$$\mathrm{M_{y,i}} = -\left(\frac{(\mathrm{z}-\mathrm{H})^2}{2}+\mu\left(\frac{\mathrm{z}^3}{6\mathrm{H}}-\frac{\mathrm{zH}}{2}+\frac{\mathrm{H}^2}{3}\right)\right)\left(\mathrm{I_{x,i}}\bar{\mathrm{q}}_\mathrm{y}+\mathrm{I_{xy,i}}\bar{\mathrm{q}}_\mathrm{x}\right)-\frac{\mathrm{I_{x,i}x_i}-\mathrm{I_{xy,i}y_i}}{\mathrm{I}_\omega}\frac{\mathrm{m_{z0}H^2}}{2}\eta_\mathrm{M} \qquad (16)$$

The first part of the equation defines bending moments that appears due to structural system bending in both directions and the second part of the equation appears due to structural system rotation.
Where η_M- represents the bending moment factor and is determined as a function of k parameter and μ- the slope of the trapezoidal load.

$$\eta_M = \frac{2}{k^2coshk}\left\{(1+\mu)\left(cosh\frac{kz}{H}-\frac{kz}{H}sinhk-coshk+ksinhk\right)\right. \\ \left.+\left(1+\frac{\mu}{2}-\frac{\mu}{k^2}\right)\left(\sinh\left(k-\frac{kz}{H}\right)+\frac{kz}{H}-k\right)k\right\} \qquad (17)$$

The maximum bending moment in case of a vertical cantilever appear at the fixed end of the equivalent column. Thus, to determine the maximum bending moment for each element, are used the above relations for z=0.

2.6. Torsional moment

The resistant elements arrangement is very important and influence significantly the torsional response of building. The building's characteristics that are influenced by the bracing elements arrangement are: the warping constant *Iw* and the radius of gyration *ip*. The high-rise structures are very sensitive to torque, to reduce torque, can choose for the arrangement of resistance elements in a way that the distance between shear center and centroid of structural system to be as small as possible; solution that leads to a symmetrical arrangement of structural elements.

The Saint Venant torsion can be applied for closed form section, obtaining good results but for open form section it must take into consideration also the warping torsion.

J-Saint Venant torsion; is based on the following assumptions:

- With constant torque a straight line on the element remains straight after the torque is applied
- The cross section is free to warp[2]

Warping torsion; this type of torsion appears to thin-walled section.

Assumptions:

- The plates which are form the cross-section deform in bending in their own planes;
- Out-of-plane bending of the plates is neglected;
- Shear deformation is neglected;
- The plates are continuously connected to each other longitudinally[2].

For most of the opened sections it must take into account both torsions: St Venant and warping torsion.

$$M_{total\ torsion} = M_{St.Venant\ torsion} + M_{warping\ torsion}$$

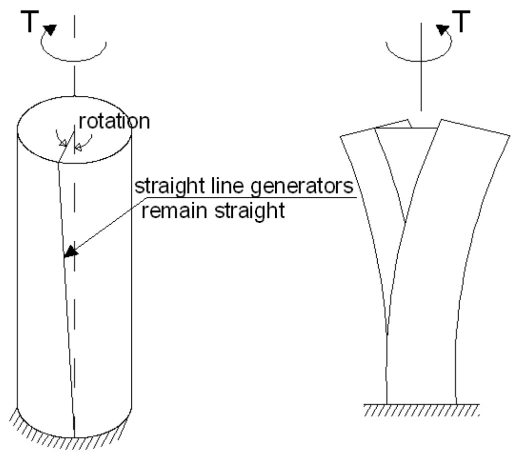


Figure 1. a) St Venant torsion b) warping torsion [2]

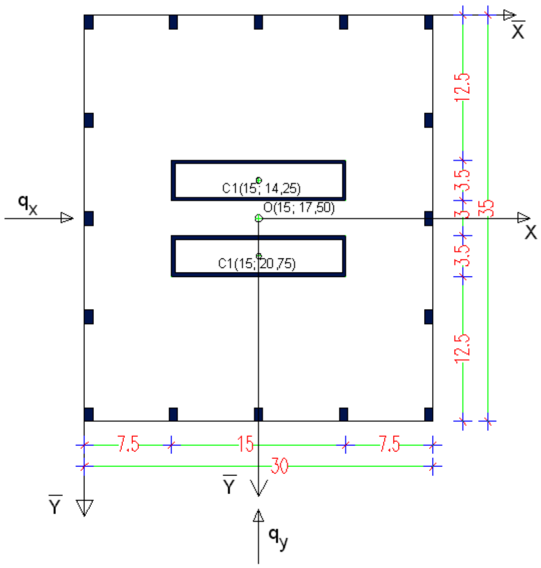
The Saint Venant torsional moment is obtained by differentiating the rotational equation once and the warping torsional moment is obtained by differentiating 3 times the rotational equation presented above.

$$M_t = m_{z0}\{H - z + \mu\left(\frac{H}{2} - \frac{z^2}{2H} - \frac{H}{k^2}\right) + \sinh\frac{kz}{H}\left(\left(1 + \frac{\mu}{2} - \frac{\mu}{k^2}\right)\sinh k + \frac{1+\mu}{k}\right)\frac{H}{\cosh k} -$$

$$-\left(1 + \frac{\mu}{2} - \frac{\mu}{k^2}\right)H\cosh\frac{kz}{H}\}$$

$$M_\omega = m_{z0}\{\frac{\mu H}{k^2} - \sinh\frac{kz}{H}\left(\left(1 + \frac{\mu}{2} - \frac{\mu}{k^2}\right)\sinh k + \frac{1+\mu}{k}\right)\frac{H}{\cosh k} + \left(1 + \frac{\mu}{2} - \frac{\mu}{k^2}\right)H\cosh\frac{kz}{H}\} \tag{18}$$

3. Numerical example



In this chapter is presented a comparative study between the results obtained using an approximate calculation method based on the equivalent column theory and the results obtained using an exact method based on the FEM.

For the approximate analysis of tall structure was developed a computer program using Matlab based on the equivalent column theory. The computer program was presented at the conference:"Structural Engineers World Congress - Como, Italy 2011"; the paper is entitled: "Structural analysis program based on the equivalent column's method".

Figure 2. Building's plan

The building has 25 levels, with a total height of H=87,5m. For this analysis model, the horizontal loads are carried entirely by the two central cores of reinforced concrete C20/25 with the modulus of elasticity $E=3,0*10^7$kN/mp and the shear modulus of elasticity $G=1,29*10^7$kN/mp. The weight per unit volume is $\gamma=0,51$kN/m^3.

The horizontal uniformly distributed load, from wind, acting on both directions is: $q_x=28$kN/m; $q_y=-24$kN/m and is represented by the concentrated forces: Fx=2450 kN and $F_y=-2100$ kN.
The input data of the computer program are the geometrical and stiffness characteristics of the central cores (table 1).

Table 1. Geometrical and stiffness characteristics of the center cores

Central cores	$\bar{x}_i$ (m)	$\bar{y}_i$ (m)	$I_{x,i}$ (m^4)	$I_{y,i}$ (m^4)	x_i (m)	y_i (m)	$I_{\omega,i}$ (m^6)	J_i (m^4)
1	15	14.25	24.327	262.76	0	-3.25	219	73.614
2	15	20.75	24.327	262.76	0	3.25	219	73.614
Σ			48.654	525.52			5988	147.228

For the approximate calculation analysis will follow the steps presented at chapter 2 and will use the relations for determining: the fundament frequency, lateral displacements in both directions, shear forces, bending moments and torsion: St.Venant and warping torsion.

For the exact calculation method based on FEM is used: Autodesk Robot Structural Analysis and ANSYS 12.1. For structural model will consider only the elements able to resist lateral loads; in this case will consider the two reinforced concrete central cores fixed at the base which are linked using rigid links. The geometrical and stiffness characteristics presented for approximate analysis will be the same for exact analysis as well. The material used: reinforced concrete C20/25 maintaining the value of modulus of elasticity longitudinal *E* and transversal *G*. Comparative results in case of uniform distribute loads in both directions: q_x=28kN/m; q_y=-24kN/m.

Table2. Maximum deformations

Calculation method	u_{max} (cm)	v_{max} (cm)	φ (rad)
Equivalent column method	1.30	12.05	0.00
F.E.M.	1.20	10.70	0.00

Table3. Natural frequency

Calculation method	f_X (Hz)	f_Y (Hz)
Equivalent column method	0.3685	1.195
F.E.M.	0.37	1.19

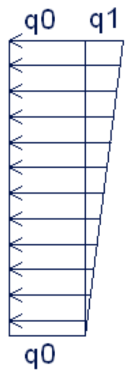
The results obtained for both central cores (shear forces and bending moments) are identical because of the building's plan symmetry.

Table4. Central core: shear forces

Calculation method		Shear force x (kN)	Shear force y (kN)
Equivalent column method		1225	1050
F.E.M.	ANSYS 12.1	1232.5	1050
	Autodesk Robot Structural Analysis	1225	1050

Table5. Central core: bending moment

Calculation method		Bending moment x (kNm)	Bending moment y (kNm)
Equivalent column method		53594	45938
F.E.M.	ANSYS 12.1	53905	45938
	Autodesk Robot Structural Analysis	53592.75	45935



Results obtain in case of a trapezoidal external load: qx_{minim}=24.50 kN/m; qx_{max}=31.50 kN/m iar qy_{min}=21 kN/m; qy_{max}=27 kN/m
To calculate the structure in case of a trapezoidal external load, will start by determining the slope coefficient: μ= q1/q0 = 0,2857.

Table6. Maximum deformations

Calculation method	umax (cm)	vmax (cm)	φ (rad)
Equivalent column method	1.37	12.75	0.00
F.E.M.	1.30	11.20	0.00

Table7. Central core: shear forces and bending moment

Calculation method	Shear force x (kN)	Shear force y (kN)
Equivalent column method	1225	1050
F.E.M.	1225	1050

Table8. Central core: bending moment

Calculation method		Bending moment x (kNm)	Bending moment y (kNm)
Equivalent column method		55826	47851
F.E.M.	Autodesk Robot Structural Analysis	55825.75	47852

Analyzing the results obtained for lateral displacements can noticed that the displacements in both directions are smaller than the maximum displacement allowed by codes H/500=17.50cm. The values of lateral displacements, fundamental frequency, shear forces and bending moments, calculated using the exact method and the approximate method of calculation are very closed, in some cases the values are identical. Thus, it can be said that the two calculation methods have been applied correctly. The same structure have been calculated for 35 floors with a total height of 122,50m and an horizontal load q_x=27 kN/m^2 and q_y=31.5 kN/m^2

The geometrical and stiffness characteristics are the same as in the cases presented above. Using the approximate calculation method, which performs a rapid analysis of structural system compared with FEM, is established that the maximum displacement v_{max}= 50.67cm is much height than the allowed limit. Thus, this structural system can not be adopted for structures with 35 floors.

4. Conclusion

The results obtained using the approximate method based on the equivalent column theory are closed to the results obtained using the exact method based on FEM, in some cases the results are even identical. The equivalent column theory is an approximate method used for comparing and checking the results obtained by FEM. Even if FEM is considered to be an exact method, may occur errors or misinterpret the results, for this reason is indicated the verification of the results using the approximate method.

It is necessary to know the approximate methods of structural analysis not only to compare and verify the exact method but also to develop new computer programs.

AKNOWLEDGEMENT
This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC**”**contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

5. References

[1] ZALKA K.A. „*Global Structural Analysis of Buildings*”, Taylor & Francis e-Library Publication, 2002

[2] Iain A. MacLeod „Analytical Modelling of Structural Systems”, Ellis Horwood Publication, 1992

[3] SMITH B.S., COULL A. ”*Tall Buildings Structures: Analysis and Design*”, Wiley-Interscience Publication, 1991

[4] TARANATH B.S., “*Reinforced Concrete Design of Tall Building*”, CRC Press, Taylor & Francis Group, 2010

[5] PETRINA M., s.a. „*Statica Constructiilor in Formulare Matriceala*”, U.T.Press Cluj-Napoca

[6] B. Rafezy, W.P. Howson, “Vibration analysis of doubly asymmetric, three-dimensional structures comprising wall and frame assemblies with variable cross-section”, Journal of Sound and Vibration, 2008, p247-267

[7] R. Hulea, B. Parv, N. Monica, M.Petrina, „Structural analysis programme based on the equivalent column’s method”, Structural Engineers World Congress, 2011.