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Assessment of a Transportation Infrastructure System using Graph Theory

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Abstract

In this paper, the topological properties of a transportation infrastructure system consisting of the highways and bridges in Newcastle County, Delaware as well as the interactions between them when subjected to disturbance were assessed. The disturbance was implemented through two disruption strategies while the response was quantified using the interaction response. Limiting values corresponding to the system response common to both strategies were obtained and used to refine the system properties until a resilient network was obtained. Thereafter, regression equations were developed for assessing the performance of the system in terms of its efficiency based on two networks selected from it. Consequently, the average efficiencies of the networks computed using these equations and based on test values were very similar and comparable to the actual network efficiency of 0.855. This showed that the regression equations are representative of the entire network and hence can be used to assess its performance.

Rezumat

În această lucrare, proprietățile topologice ale unui sistem de infrastructură de transport format din autostrăzi și poduri în Newcastle, Delaware precum și interacțiunile dintre ele atunci când sunt supuse la perturbări au fost evaluate. Perturbare a fost pusă în aplicare prin intermediul a doua strategii de întrerupere în timp ce răspunsul a fost cuantificat cu ajutorul interacțiunii răspuns. Valorile limită corespunzătoare răspunsului sistemului comun pentru ambele strategii au fost obținute și folosite pentru a rafina proprietățile sistemului până la obtinerea unei rețele de rezistentă. Ulterior, ecuațiile de regresie au fost dezvoltate pentru evaluarea performanței sistemului în ceea ce privește eficiența acestuia pe baza a două rețele selectate. Prin urmare, eficiența medie a rețelelor calculate folosind aceste ecuații bazate pe valorile de încercare au fost foarte asemănătoare și comparabilă cu eficiența rețelei actuale de 0.855. Aceasta a arătat că ecuațiile de regresie sunt reprezentative pentru întreaga rețea și, prin urmare, pot fi utilizate pentru a evalua performanțele sale.

Keywords: Topological properties, disruption strategies, interaction response, resilient network, regression equations.

1. Introduction

One of the most critical infrastructures necessary for the smooth functioning of our modern day society is the transportation system. Its components link together to form a network whose primary

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function is to meet the requirements of the society such as movement of goods and services, power generation, food security and financial sustainability [1]. Hence, there is a direct or indirect interaction between these elements such that failure in one when it is overloaded can lead to failure in others. A situation in which the failure is propagated throughout the network is referred to as cascading failure and the system is referred to as a critical infrastructure.

The degree of interaction between infrastructural components strongly influences their operational characteristics which can create subtle interactions and feedback mechanisms that often lead to fatal consequences during disasters [2]. A number of graphical models in which the system components are represented by nodes and the interactions between them are represented by edges have been established to provide a logical explanation of the interactions that exist between systems. Such models include regular, random and small world network models.

This study provides a novel approach for graphical analysis of the complex network of highways and bridges in Newcastle County, Delaware based on their topological properties. This is unlike previous studies which have focused on the analysis of road networks and their corresponding intersections/junctions. This study also develops a procedure that evaluates the characteristics of the complex system and their interactions using network topological characteristics, assesses the effects of the bridge and highways network size and topology on its characteristics, studies the impact of disruption on the network and the interactions between the components, develops equations for assessing and predicting the topology of highway and bridge networks and develops equations for predicting the interactions based on efficient network growth patterns with ability for resilience.

2. Background history of network models

Initially, real life networks were analyzed as regular graphs until the late 1950's when large networks with no defined organizational patterns began to be described as random graphs. In 1959, two Hungarian mathematicians, Paul Erdos and Alfred Renyi modeled networks graphically by randomly connecting the nodes with links [3]. It was based on the hypothesis that despite the random inclusion of links, the resulting graph will be largely conservative meaning that on average, the probability of the nodes having the same number of links followed a Poisson distribution. Hence, random networks are referred to as scaled networks, a framework upon which complex systems were analyzed thereafter because it was the only suitable method available in the absence of better computer technology. By the late 1990's, rapid progress began to be made in complex network analysis with the aid of super computers which made analysis of vast data quantities and their conversion into databases possible. Further investigations showed that the topology of real large networks such as the internet substantially differed from the topology of random graphs as produced by Erdos and Renyi, therefore new methods, tools and models needed to be developed.
Subsequent research work on networks revealed that real systems had properties lying between those of regular and random networks [4]. This group of networks was called small world networks which in simple terms described the fact that despite the large size of most networks, there is a relatively short path between the components called the characteristic path length. They also tended to cluster in which several nodes form cliques by linking to a common node.

More recent studies of small world networks revealed three types namely scale-free networks broad-scale networks and single-scale networks [5]. Scale free networks seem to have generated the greatest interest in research circles because they effectively explain cascading failure in systems. They were developed by Reka and Barabasi in 1999 at the University of Notre Dame. Their original intention was to determine the structure of the World Wide Web (WWW) on the premise that scale free networks would share similarities with random networks where the links would be evenly distributed amongst the pages. However, they discovered that several of the pages had very few links while a very few number of pages had thousands of links such that the networks have no scale. Lately, these networks have been developed for characterizing urban street networks based on measurements of two sets of properties [6]. The first set includes the shortest path distance and clustering properties. The second set is related to the node's centrality measure such as the vertex or edge betweenness centrality and edge information centrality.

3. Graph theory: definitions and notations

Graph theory is used as the analytical framework in which the system is represented as a graph, G consisting of nodes or vertices which belong to the set of elements V connected by links or edges which belong to the set of elements E. The nodes represent the fundamental units or components of a system while the links represent the interactions between them. Therefore, a graph, $G = \{V, E\}$ consists of a set of elements V and E where $V \neq 0$ and E is a set of pairs of elements of V. The set of elements in $V = \{v_1, v_2, \ldots, v_n\}$ and in $E = \{e_1, e_2, \ldots, e_n\}$. An edge, e_i is directed if it runs in one direction only and undirected if it runs in both directions. A graph, G is directed if all of its edges are directed, however, it is undirected if each edge between two nodes runs in two opposite directions and usually no arrows are included in this case. The number of nodes in G is called the order, N while the number of links is called the size, M.

A node is usually identified by the symbol i while each of the links is identified by the nodes i and j that it connects as e_{ij} and as such the link is said to be incident on the two nodes. Two nodes joined by a link are called adjacent or neighbor nodes. In an undirected graph, the order of the nodes is not significant because $e_{ij} = e_{ji}$ while the number of edges, M is at least zero for an isolated node and at most $N(N-1)/2$ for a complete graph in which all the nodes are connected. The geodesic path between two nodes is the shortest path through the network from one node to the other where it is possible to have more than one such path between two nodes [7] while the average shortest path length is the mean of the lengths of all the shortest paths between any two nodes i and j [8].

A graph is represented by the adjacency matrix, W which is a product of an N x N adjacency matrix A whose entry a_{ij} is equal to 1 when there is an edge between nodes i and j and 0 otherwise and another N x N matrix, L whose every entry l_{ij} is equal to the length of the shortest distance connecting nodes i and j [9]. W is expressed in equation 1 and it considers both the geometry and the spatial location of the network.

$$W = \sum_{i,j} a_{ij} l_{ij} \tag{1}$$

4. Network topological properties

The network topological properties assessed include average shortest path distance (ASpd), average clustering coefficients (ACc), network density (Nd), average vertex degree (AVd), node (Nbc) and link (Lbc) betweenness centralities, network connectivity loss (NtWkcl) and efficiency (Eff).

4.1 Vertex degree, vertex degree distribution and vertex degree ranking

Vertex degree is the number of edges incident on a node which in terms of the adjacency matrix, is given as:

$$k_i = \sum_{j \in N} a_{ij} \tag{2}$$

In a directed graph, each node degree has two components namely the number of outgoing links, $k^{out} = \sum_{j \in N} a_{ij}$ and the number of in-coming links, $k^{in} = \sum_{j \in N} a_{ij}$. The sum of the two, that is, $k = k_i^{in} + k_i^{out}$ gives the vertex degree of the node. The average vertex degree, <*k*> of *G* is given as:

$$< k > = \frac{1}{N} \sum_{v \in N} d(v_i) = \frac{1}{N} \sum_{v \in N} k_i \qquad (3)$$

where $d(v_i)$ is the vertex degree over each node.

The spread of node degree across the network is identified by the vertex degree distribution function *P(k)* that gives the probability that any node selected at random will have exactly *k* edges. The fundamental topological characteristics of a graph, *G* can be obtained in terms of the degree distribution [10]. If *P(k)* follows a Poisson distribution with short thin tails, then the network is equally vulnerable to random perturbations since all the vertices have a typical average degree. However, if *P(k)* follows a power law with long thick tails, then the networks show a reasonable level of resilience to random perturbations [11]. Vertex degree ranking is the process by which the nodes in a network graph are rank ordered, based on their vertex degree *d(v)* or *k*, from the one with the highest frequency of attachments to the one with the lowest frequency.

4.2 Shortest path lengths

The shortest path is defined as the quickest route from one point to another within the same network. Normally, all the shortest paths of a graph *G* are represented in matrix form such that the entry d_{ij} gives the geodesic or actual distance from node *i* to node *j*. The average separation between two nodes in the graph is given by the average shortest path length, *L*. '*L'* can be regarded as a global indicator of network connectivity obtained by calculating the mean of geodesic lengths over all pairs of nodes and is given by:

$$L = \frac{1}{N(N+1)} \sum_{\substack{i,j \in N \\ i \neq j}} d_{ij} \qquad (4)$$

where *N* is the order of the graph.

4.3 Clustering and the clustering coefficient

Networks have a high tendency to form cliques of nodes through clustering and for a graph *G*, the degree of clustering is measured by a quantity called the clustering coefficient C_C given by :

$$C_c = < c > = \frac{1}{N} \sum_{i \in N} c_i \qquad (5)$$

where c_i is the clustering coefficient of each node or vertex *v*. The quantity c_i is calculated as the ratio between the actual number of edges e_i in the subgraph G_i of the neighbors of *v* and $k_i(k_i-1)/2$, the maximum possible total number of edges in G_i ;

$$c_i = 2e_i/[k_i(k_i-1)] \qquad (6)$$

Clustering coefficient is regarded as a local measure of connectivity since it measures the level of connectedness of the network over very short path lengths.

4.4 Vertex and edge betweenness

The betweenness b_i of a node *v* is defined as the total number of shortest paths that pass through it when the shortest paths are calculated between every pair of nodes $(i, j) \in V(G)$ and *v* is not an end of any shortest path [11]. It quantifies the importance of a node to a network and is used for obtaining the flow through intermediate nodes. Hence, it accounts for both the topological and dynamic properties of the network. It is given by:

$$b_i = \frac{1}{(N-1)(N-2)} \sum_{\substack{i,j \in N \\ i \neq j}} \frac{n_{jk}(i)}{n_{jk}} \qquad (7)$$

where n_{jk} is the number of shortest paths connecting *j* and *k* and $n_{jk}(i)$ is the number of shortest paths connecting *j* and *k* passing through node *i*. Similarly, the edge betweenness is defined as the number of shortest paths between pairs of nodes that run through an edge.

4.5 Efficiency and connectivity loss

Two of the parameters used for measuring system performance before or after disruptions are the efficiency and the connectivity loss. Based on the assumption that flow between two nodes, *i* and *j* occurs through the shortest path [12], the relationship for the average efficiency, *E* of a network graph, *G* is given as:

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \in_{ij} \qquad (8)$$

where $\in_{ij}$ is the efficiency matrix of communications between all pairs of vertices *i* and *j* calculated as the inverse of the actual shortest distances between them, that is $\in_{ij} = 1/d_{ij}$. *E(G)* was thereafter normalized by the optimal efficiency of the complete graph, G_{id} as given in equation 9 in which all the nodes are connected by edges *N(N-1)/2* [13].

$$E(G_{id}) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{l_{ij}} \qquad (9)$$

where l_{ij} is the Euclidean distance between the nodes and $d_{ij} \geq l_{ij}$. The normalized efficiency is given as:

$$E = \frac{E(G)}{E(G_{id})} = \frac{\sum_{i \neq j \in G} \frac{1}{d_{ij}}}{\sum_{i \neq j \in G} \frac{1}{l_{ij}}} \qquad (10)$$

where $0 \leq E \leq 1$. *E(G)* is considered as a global measure of network efficiency while *E* can be used to assess both the global and local levels of efficiency.

Network Connectivity Loss, C_l is a parameter used for quantifying the average decrease in the number of generating nodes with connecting paths to distribution nodes in a system. C_l was calculated by using the equation [14]:

$$C_l = 1 - \left\langle \frac{N^i}{N} \right\rangle_i \qquad (11)$$

where C_l is averaged over all the nodes, N is the initial number of properly functioning nodes and

N^{i} is the number of remaining nodes in the network.

5. Network models

Three major network models have been developed with varying topology to characterize real world networks namely regular, random and small worlds networks depending on the probability of connectedness, *p* of the nodes.

5.1 Regular network models

A regular network is a graph in which each node has exactly the same number of links to its neighboring nodes. It has long path lengths, hence, they are unsuitable for modeling real networks despite the fact that they have high clustering coefficients. Examples of regular networks include regular grids in which all the nodes have exactly four links and hexagonal lattices where each node is connected to three other nodes. In regular networks, the probability of connectedness of the nodes, *p* approaches zero.

5.2 Random Network Models

A random network graph is one that possesses both short path lengths and low clustering coefficients typical of a lot of real life networks such as urban streets and intersections. It is generated by randomly connecting *N* number of nodes with *E* links each with a probability of *p*. The network is completely connected with the maximum possible number of edges given as *N(N-1)/2* if p = 1. The main objective of random network graph studies was to determine at what connection probability, *p* distinct properties of a graph will manifest.

5.3 Small world network models

Small world networks exhibit topological properties intermediate between those of regular and random networks which interestingly, also characterize real life networks. In effect, they manifest high clustering coefficients and short path lengths. Small world networks were developed by rewiring each of the edges of a regular network at random with a probability *p*. The process involved shifting one end of the connection to a new node selected at random from the whole network with a constraint that any two different vertices cannot have more than one link between them. The small-world property results from the immediate drop in the characteristic path length as soon as *p* is slightly larger than zero. This is because the rewiring process creates long-range edges acting as shortcuts which connect distant nodes.

5.4 Scale free network models

Scale free networks are a class of small world networks in which there exists a few nodes called hubs to which large numbers of other nodes are attached and a considerable high number of nodes to which only a few nodes are attached. This creates an uneven distribution of connectedness across the network such that the hubs exhibit a power law of the form $P(k) \sim k^{-\lambda}$ where λ varies between 2 and 3.

Hence, the average path length of the scale free model of the same size and the same average number of links as a random model is smaller while the clustering coefficient is much higher leading to heavy-tailed degree distribution. This property makes them error tolerant or resistant to random failure but highly vulnerable to coordinated attacks where error tolerance is defined as the presence of a giant cluster in a network even after a fraction of the nodes have been removed [15]. Interestingly, this same advantage turns out to be a disadvantage because any attack targeted at any of the hubs could lead to rapid disintegration of the network.

6. Interaction of System components

The degree of interaction between the two sets of components comprising a transportation infrastructure system significantly influences its operational characteristics. This interaction is described as a relationship in which the components are dependent on one another. The levels of dependency or coupling can be classified into four types namely physical interaction in which the states of two or more infrastructure components depend on the material outputs of the others, logical if their states are based on human decisions and actions, cyber if the states depend on the information transmitted between them and geographic if the interaction occurs when the infrastructures affect one another based on their spatial location and proximity to each other rather than on their states [16].

The interactions make it impossible to analyze the behavior of the infrastructure in isolation of its environment. The result is a higher coupling effect between the systems that makes them more vulnerable to cascading failures [17]. Such failure can be mitigated by introducing resilience into the system whereby it can recover from a natural or intentional hazard, restructure itself and continue to perform effectively.

6.1 Disruption mechanisms and the interaction response

Disruption in networks is simulated by two node removal strategies namely the random node removal strategy (RNRS) and the targeted node removal strategy (TNRS). The interaction response of the network to disruption is quantified as a function of their strength of coupling and is determined for each topological property as:

$$\therefore I_r = \left| 1 - \left(\frac{\text{Magnitude of topological property after perturbation}}{\text{Magnitude of topological property prior to perturbation}} \right) \right| \tag{12}$$

A resilient system is then obtained as a compromise between the two strategies at their points of equilibrium.

7. Highway and bridge networks

A lot of literature is available on studies carried out to understand the nature of highway networks in the United States. In this paper, the highway system of Newcastle County in Delaware as well as the interconnecting bridges is studied and developed into a network that is analyzed using graph theory. The bridges provide connectivity in the highway network within a particular geographic region. Hence the bridges are represented by nodes while the highways are represented by the links or edges.

Highway networks in the United States have been shown in previous studies to be exponentially distributed and capable of being represented by random networks [18]. However, most previous works did not consider bridges as the highway connectors; rather the highway intersections were considered as the connectors and represented by nodes while the routes branching off from them were considered as links. Considering the bridges as nodes in this case may cause the network to deviate from a random state.

The main objective is to develop a procedure for assessing the characteristics of a transportation infrastructure system and its component interactions using graph theory. Sub-objectives are to study the impact of disruption on the network and to develop equations for assessing the performance of the system with capacity for resilience.

7.1 Network analysis

A total of 1310 bridges and 27,836 highway/road segments were identified for this study (using ArcGIS 9.2 software package) in the shapefile data obtained from the Delaware Department of Transportation (DELDOT) for Newcastle County, Delaware. Analysis was carried out on two representative sample data sets selected from two different locations in the county with each measuring 16 km^2. The second sample served as a check on the analysis results of the first set.

The two sample data sets consisting of 21 and 32 bridges respectively were developed into two network case studies based on their geographical location. The process involved connecting the bridges by weighted links where the weights corresponded to the shortest path lengths between pairs of bridges. Figure 1 shows the first network case study for data selected from North-West of Newcastle County with 21 bridges and 29 links. Thereafter, adjacency matrices for both networks were developed according to equation 1. Disruption in the network was implemented by using the two node removal strategies. Each new network that emerged after each node removal was analyzed to determine its properties which gave a set of data that was combined and plotted for each property.

Figure 2 shows the graphs of the average shortest path distances for the initial network of the first case study under the TNRS and RNRS. The point of convergence of the graphs at ASpd of 2.803 and node removal fraction of 0.1333 was selected as the limiting value for refining the network since the system responds the same way to the two disturbances at this point. The node removal process was repeated with the limiting value applied to *W*, the properties were analyzed and the new corresponding graphs were plotted.

The process was repeated until the results became consistent for each property and the convergence point remained unchanged resulting in the final network. The corresponding interaction response graphs for the ASpd of the initial network were also obtained.

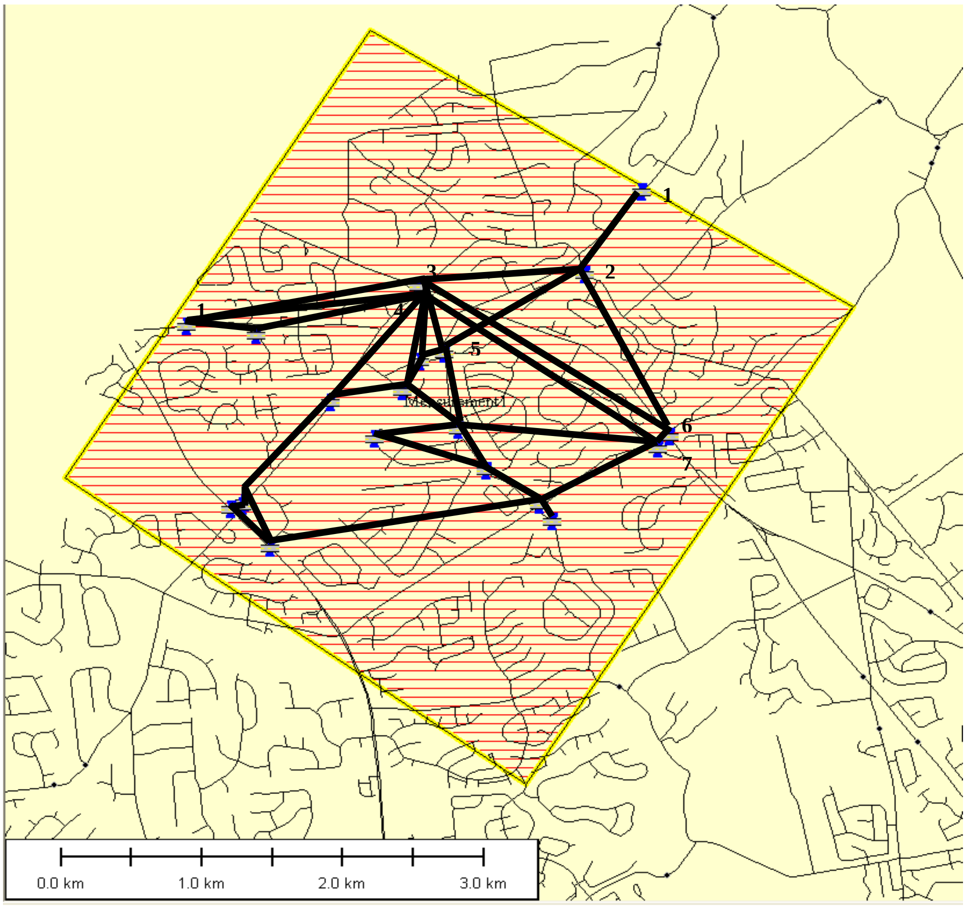

Figure 1. First Network Case Study for data selected from North-West of Newcastle County (consisting of 21bridges and 29 highway links)

Figure 3 shows the graphs of the average shortest path distances under the TNRS and RNRS for the final network obtained after the refinement process in which the convergence point occurs to a value of average shortest path distance of 2.3107 at a node removal fraction of 0.236. The corresponding interaction response graphs for the ASpd of the final network were also obtained. Table 1 compares the magnitude of the differences in area under the graphs of the TNRS and RNRS of both the initial and final network properties and interaction response. Similar analysis was carried out for the second case study.

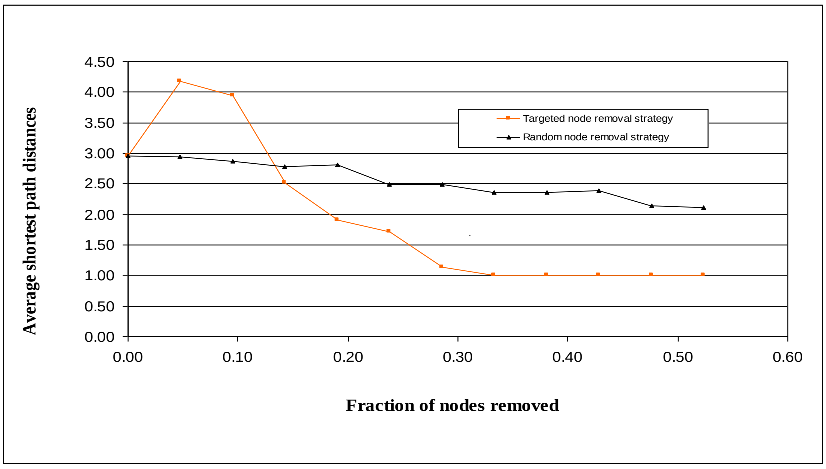

Figure 2. Graph of average shortest path distances against fraction of nodes removed (Initial network)

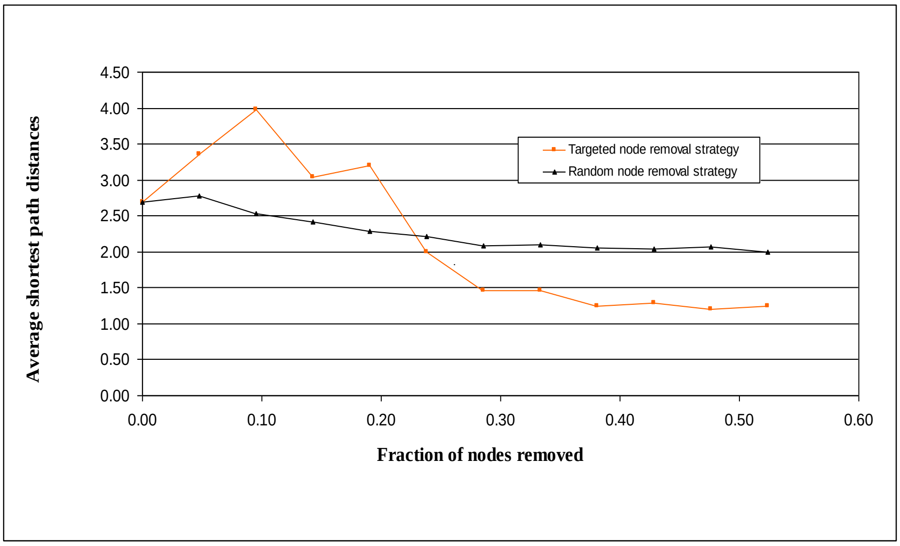

Figure 3. Graph of average shortest path distances against fraction of nodes removed (Final Network)

Table 1: Comparison of values of the initial and final network properties and interaction response

Network properties					
S/N	Property	Initial (1)	Final (2)	Difference $\lvert(1)-(2)\rvert$	Diff. (%)
1	AVd	0.356	0.2525	0.1035	29.07
2	Nd	0.0563	0.0405	0.0158	28.06
3	ASpd	0.2810	0.005	0.276	98.22
4	Acc	0.1295	0.0468	0.0827	63.86
5	Nbc	0.0411	0.0116	0.0295	71.78
6	Lbc	3.2924	0.8125	2.4799	75.32
7	NtWkcl	0.3347	0.1887	0.146	46.62
8	Eff	0.2363	0.1063	0.130	55.01

Interaction Response (I_r)					
S/N	Interaction Response	Initial (1)	Final (2)	Difference $\lvert(1)-(2)\rvert$	Diff. (%)
1	I_r(AVd)	0.266	0.1881	0.0779	29.29
2	I_r(Nd)	0.1577	0.0725	0.0852	54.03
3	I_r(ASpd)	0.1716	0.1163	0.0553	32.23
4	I_r(ACc)	0.3488	0.4139	0.0651	18.66
5	I_r(Eff)	0.2617	0.1210	0.1407	53.76

7.2 Development of regression equations

Regression analysis was carried out on the combined values of the final network properties for each case study to give equations of the system efficiency and interaction response. The analysis is based on nonlinear equations of the form in equation 13 [19].

$$y_i = \beta_o + \beta_1 x_i + \beta_2 x_i^{\,2} + ...+ \beta_r x_i^{\,r} + \varepsilon_i \qquad (13)$$

Tables 2 and 3 show the regression equations derived for the topological properties and interaction response of the network case study respectively with the corresponding *R* squared values. The equations are verified by using test values corresponding to properties of the system at a particular stage of node removal analysis whose corresponding efficiency value is 0.855. Also, similar equations were derived for the second case study.

Table 2: Regression equations for the network topological properties of the 1st network case study

S/N	Property	R^2 values	Test value	Regression Equations	Calc. Eff. (2)	Difference $\|(1)-(2)\|$	Diff. %
1	AVd	0.891	1.176	Eff = -0.132+1.718*AVd-0.727*AVd^2	0.883	0.028	3.28
2	ASpd	0.961	2.287	Eff = -0.510+0.927*ASpd-0.146*ASpd^2	0.846	0.009	1.05
3	Nbc	0.880	0.059	Eff = 0.447+8.320*Nbc-35.781*Nbc^2	0.813	0.042	4.91
4	Lbc	0.860	10.75	Eff = 0.405+5.196E-02*Lbc-1.151E-03*Lbc^2	0.831	0.024	2.81
5	NtWkcl	0.885	0.309	Eff = 0.885+0.152*NtWkcl-0.689 *NtWkcl^2	0.866	0.011	1.29
6	Eff	0.992	0.855 (1)	Eff = -.870+0.692*ASpd+0.395*AVd +1.967*Nbc -3.315E- 3*Lbc+0.468 *NtWkcl-0.107*ASpd^2+5.041*Nbc^2	0.860	0.005	0.58

Table 3: Regression equations for the interaction response of the 1st network case study

S/N	Interaction Response	R^2 values	Test value	Regression Equations	Result
1	*Ir*(AVd)	1.000	1.176	I_r(Avd) = 1-0.724*Avd	0.149
2	*Ir*(Nd)	0.934	0.147	I_r(Nd) = 1.839-25.086*Nd+89.316*Nd^2	0.081
3	*Ir*(ASpd)	0.931	2.287	I_r(ASpd) = 1.698-1.183*ASpd+0.220*ASpd^2	0.143
4	*Ir*(ACc)	0.993	0.087	I_r(ACc) = 1.001-18.518*ACc+94.737*ACc^2	0.107
5	*Ir*(Eff)	0.995	0.855	I_r(Eff) = 1.100-1.474*Eff+0.304*Eff^2	0.062

8. Discussion

Figure 2 shows that the shortest path distances seem fairly consistent under the RNRS but irregular under the TNRS. At this juncture, it might be convenient and impulsive to argue that the system should be represented by a random network but random networks cannot resist intentional attacks. Hence, a combination of the effect of both strategies must be considered to arrive at a resilient network.

Consequently, a state of equilibrium is reached at 0.1333 node removal fraction with a corresponding ASpd value of 2.803.This means that the system is capable of resisting both disturbance types at this point. So, using this as the limiting value and the process discussed in section 6.2, the system is refined until a final network is obtained in which the difference in area of the ASpd graphs under both strategies is reduced (figure 3) when compared with that of the difference in area under both strategies in figure 2. Consequently, the lower the value of this difference, the more the graphs under the TNRS and RNRS converge and likewise the higher the level of resilience introduced in the network. This also translates into reduced disruption in the final network as compared with that of the initial network.

In table 1, the highest convergence towards a resilient network is achieved with respect to the ASpd property to a value of 98.22% and the Interaction response of the Nd to a value of 54.03%. This shows the ASpd as the best parameter for assessing the network geometry and the network density as the best for understanding how the system responds to disruption. Also, table 1 shows that the difference in areas of graphs generated under the two node removal strategies for both the properties and the interaction response drops from the initial to the final network except in the case of the ACc (which is highlighted).
Table 2 shows the regression equations derived for the network efficiency based on five properties

whose R^2 values were not less than 0.85. The remaining properties give R^2 values much lower than 0.85 and so are not considered. It can be observed that the equations give efficiency values very close to the highlighted target value of 0.855 based on the test values used. Hence, the differences between the calculated efficiencies and the target efficiency are not significant. This difference is expressed as a percentage of the target efficiency to give the % difference in efficiency indicating the level of accuracy achieved. In this case, the accuracy level is high since the values are very low with none exceeding 5%.

Lastly, table 3 shows regression equations for the interaction response (I_r) also based on five properties whose R^2 values were not less than 0.85. The results were verified using the test values and they indicate very low disruption in the final network under the node removal strategies (with no value exceeding 0.15). All the analysis results obtained for the first network case study were verified by those of the second case study.

9. Conclusion

The main contribution of this paper is the development of an objective procedure that can be applied to any infrastructure system to determine its characteristics without excessive computing effort and time. However, the regression equations developed are peculiar to the system studied in this paper and cannot be applied directly to any other system.

In particular, this procedure can be useful in bridges and highways system planning to prioritize maintenance activities at the network level. The process will require selecting a small section of the system, analyzing it for the properties and substituting the values in the regression equations to give the efficiency and interaction response of the entire system.

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