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Flat slabs with spherical voids. Part I: Prescriptions for flexural and shear design

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Abstract

Much of the volume of concrete placed in any civil structure is given by the floor system(s) used. The advent of new solutions that incorporate spherical hollow cores create the premises for providing adequate guidelines to be used by practicians, suitable to accommodations for a variety of onsite conditions. The here-in research programme is aimed at answering the fundamental question of structural behaviour for such floor systems, subjected to flexure and shear, as well as their limits of use.

Rezumat

Mare parte din volumul de beton pus în operă într-o structură civilă este reprezentat de sistemul de planşeu utilizat. Apariţia unor soluţii care încorporează goluri sferice creează premisele pentru a furniza practicienilor de pretutindeni îndrumări clare, adaptabile situaţiilor concrete din practică. Studiul prezentat în continuare doreşte să răspundă unor întrebări fundamentale, precum comportarea structurală sau limitele utilizării acestora.

Keywords: concrete slab, flat plate, spherical hollow cores, commercial solutions, shear strength

1. Introduction

Among the cast-in-place floor systems in use for multistorey high structures, flat plates are commonly used based on the ability to fulfil technical and functional requirements as well as the increased speed of construction. Since dead loads (own weight) of the floor system increases as openings are increasing, the middle height of the cross section can be replaced (frequently) by (plastic spherical) hollow cores (Fig. 1), as it does not contribute to the resistive capacity or the durability of the member. This solution allows the floor system to work two-ways while reducing own dead load.

Some of the advantages of using a spherical hollow core flat plate as compared to the full flat plate are:

- Reduction in the volume of materials used in the structure (both concrete and steel) not just for the floor themselves;
- Reduction of labour costs and labour time for all trades involved in placing reinforcing bars;

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- Increase openings while decreasing the no. of columns, providing thus improved architectural floor layout and more clear volume;
- Reduction of the carbon dioxide footprint created by industrial activities associated with its production;
- Achieve sustainability by use of recycled plastic materials for the hollow cores;

Some disadvantages, inherent to any hollow core system, that impair the use in actual practice are:

- Reduction of sectional rigidity;
- Reduction of punching shear capacity;
- Reduction of shear capacity;

Since the advantages may be considered to out-weight the disadvantages, it may be appropriate to research the applicability of this floor system as well as its limits, considering only the gravitational loads applied.

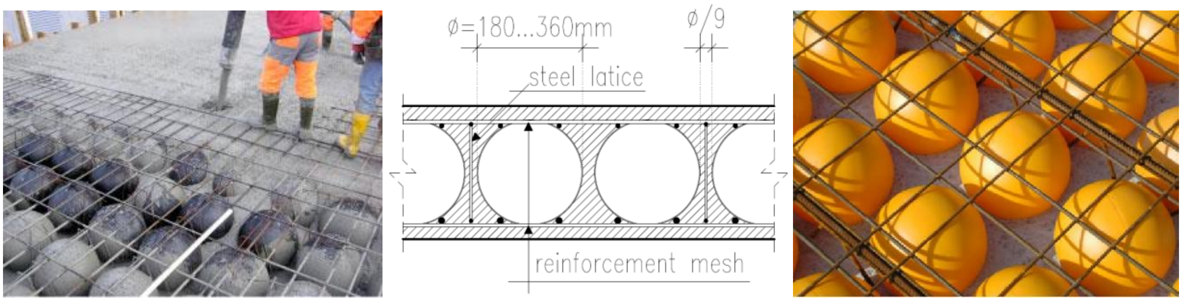

Figure 1. Details for the spherical hollow cores flat slab system [2] [1].

2. Generic details on loading levels

In terms of maximum forces, flat plates develop maximum bending moments in edge strips at joints with supporting columns as well as negative bending for the slab. Such areas are subjected also to punching shear, therefore the hollow cores should be eliminated from the cross section around the head of the column. In turn, the negative bending that develops around the resting areas on the columns for a spherical hollow core flat plate is identical in its characteristics with a full flat plate.

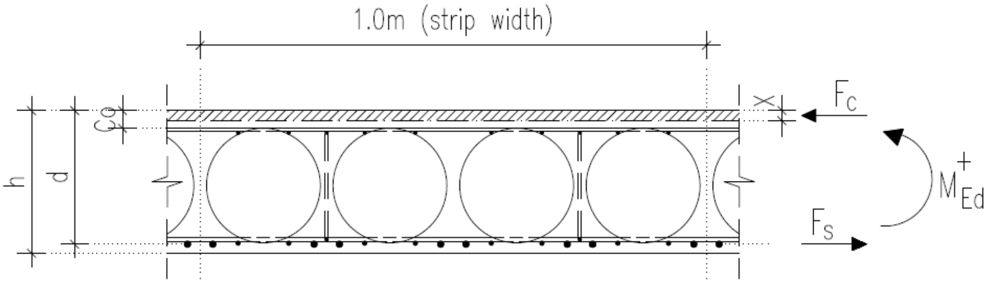

Figure 2. Main parameters for flexural design

According to EC2 [3] the compressive stress block for concrete may be assumed to be rectangular. As long as the neutral axis lies above the upper limit of the hollow cores (top concrete area) there is no difference in terms of flexural design in-between the hollow and full flat plate (see Figure 2, illustrating the case $x \le c_0$ with c_0 the concrete cover for the plastic cores). When the neutral axis lies lower so that it intersects the hollow cores, it is assumed that concrete surrounding the hollow cores can work spatially (the concrete strut varies from full height of the section – in-between two

consecutive hollow cores, to minimum – for the vertical diameter passing through the spheres).
It is therefore mandatory to study the parametric evolution of the neutral axis, for the inner middle span of an edge strip, for various load levels using approximate calculations methods, such as the “coefficients method” presented in [4] assuming a uniform layout for the supporting columns (square bay). The parameters tested are:

- openings: L=6.0÷14.0 m;
- load level: q_k=2.5÷10.0 kN/m^2.

Tables 1 and 2 present the results of this analysis, while Figures 3 and 4 show the variation of the neutral axis as a function of the fore-mentioned parameters.

Table 1: The neutral axis position for various load levels (inner panel)

L	h_{nec}	h_{eff}	c_o	q_k=5.0 kPa			q_k=10.0 kPa		
[m]	[mm]	[mm]	[mm]	p [%]	x [mm]	c_o/x	p [%]	x [mm]	c_o/x
6.0	200.0	230	30	0.17	14.0	2.14	0.25	21.0	1.43
7.0	233.3	230	30	0.23	19.3	1.56	0.35	29.0	1.03
8.0	266.7	280	35	0.21	21.7	1.61	0.31	31.8	1.10
9.0	300.0	340	50	0.19	24.4	2.05	0.27	34.7	1.44
10.0	333.3	340	50	0.24	30.0	1.67	0.34	42.8	1.17
11.0	366.7	390	55	0.23	33.7	1.63	0.32	47.2	1.17
12.0	400.0	450	70	0.21	36.6	1.91	0.29	50.1	1.40
13.0	433.3	450	70	0.25	43.6	1.61	0.35	59.8	1.17
14.0	466.7	450	70	0.30	51.2	1.37	0.41	70.5	0.99

Table 2: The neutral axis position for various load levels (end panel)

L	h_{nec}	h_{eff}	c_o	q_k=5.0 kPa			q_k=10.0 kPa		
[m]	[mm]	[mm]	[mm]	p [%]	x [mm]	$x<c_o$?	p [%]	x [mm]	$x<c_o$?
6.0	200.0	230	30	0.31	25.8	1.16	0.47	39.1	0.77
7.0	233.3	230	30	0.43	35.8	0.84	0.67	54.9	0.55
8.0	266.7	280	35	0.39	40.2	0.87	0.58	59.8	0.59
9.0	300.0	340	50	0.35	45.0	0.78	0.51	64.9	0.54
10.0	333.3	340	50	0.44	55.7	0.9	0.64	80.8	0.62
11.0	366.7	390	55	0.42	62.5	0.88	0.60	88.8	0.62
12.0	400.0	450	70	0.39	67.8	1.03	0.55	93.9	0.75
13.0	433.3	450	70	0.47	81.2	0.86	0.66	113.2	0.62
14.0	466.7	450	70	0.56	97.1	0.73	0.78	135.0	0.52

It may be concluded that for inner panels the percentage of reinforcement is less than 0.50% for all load levels, while for end panels this threshold is not respected only for load levels greater than 5.0 kN/m^2. In the case of increased load levels, the reinforcement percentages increase but are still below 0.8%.

The neutral axis position is different for inner and end panels only for load levels bigger than 5.0 kN/m^2. Below this threshold, the neutral axis is always above the hollow cores for all load levels in all inner panels, while in end panels, the neutral axis lies somewhere at the hollow core level only for load levels greater than 5.0 kN/m^2.

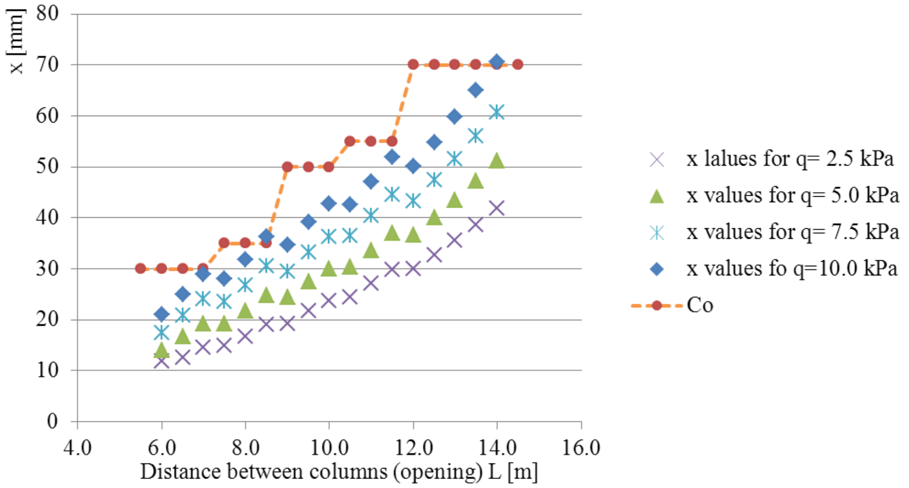

Figure 3. Inner panel – neutral axis variations.

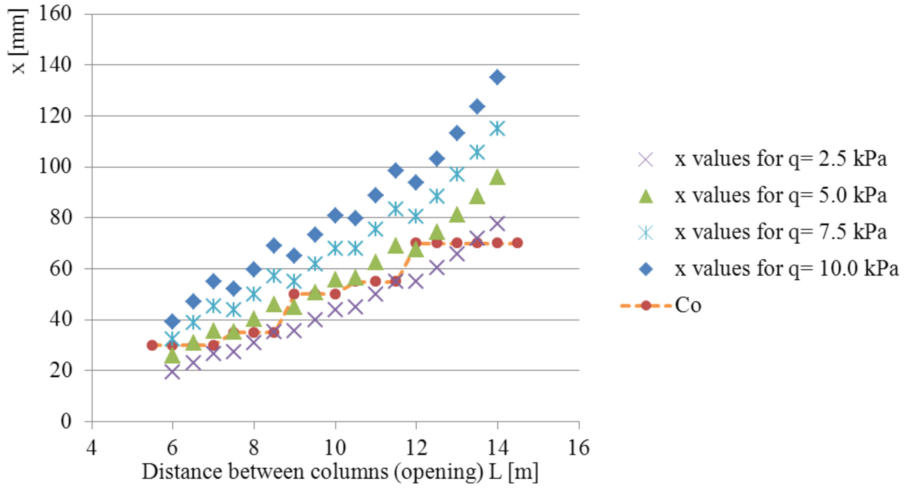

Figure 4. End panel – neutral axis variations.

3. Shear design for spherical hollow cores flat plates

In terms of flexural design, as long as live loads fulfil the 5.0 kN/m^2 threshold, there are no differences as compared to the full flat plate. The situation is very much different in terms of shear design, since the presence of hollow cores changes the behaviour of the member. Several models are presented next, with special emphases on EC2 [3] and ModelCode2010 [5].

3.1 According to EC2 [2], for no shear reinforcement

For a strip of 1 m in width:

$$V_{Rd,c} = \left[C_{Ed,c} \cdot \eta_1 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d \qquad (1)$$

But not less than $V_{Rd,c} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (2)

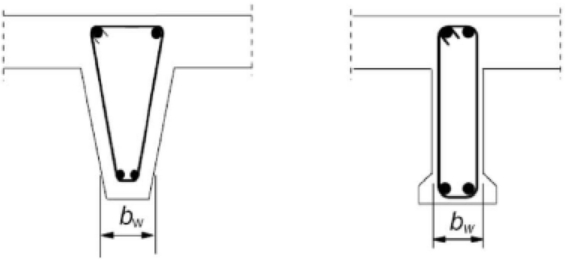

Figure 5. Minimum width b_w according to EC2 [3].

3.2 According to Model Code 2010 [4], for no shear reinforcement

Several models may be used depending on the complexity and therefore the results intended MC 2010 [5]. The resistive capacity of concrete is:

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot z \cdot b_w \tag{3}$$

with a maximum value of 8 MPa for $\sqrt{f_{ck}}$. The k_v coefficient may be determined for three degrees of approximation as:

$$k_v = \frac{200}{(1000 + 1.3z)} \leq 0.15 \text{ first degree} \tag{4}$$

$$k_v = 0 \quad \text{second degree} \tag{5}$$

$$k_v = \frac{0.4}{(1 + 1500\varepsilon_x)} \cdot \frac{1300}{(1000 + 0.7k_{dg} \cdot z)} \leq 0.15 \text{ third degree} \tag{6}$$

with

ε_x is the longitudinal strain in concrete measured at half the depth of the cross section

$$\varepsilon_x = \frac{M_{Ed}/z + V_{Ed} + 0.5N_{Ed} - A_p \cdot f_{p0}}{2 \cdot (E_s A_s + E_p A_p)}, \tag{7}$$

k_{dg} is a coefficient taking into account the maximum aggregate size

$$k_{dg} = \frac{48}{16 + d_g} \geq 1.15 \tag{8}$$

f_{p0} is the stress for tendons when the corresponding level concrete has nil strains
d_g is the maximum aggregate size

It may be concluded that previously cited references have in common the use of the minimum width in tension b_w. Considering the situation of a section plan placed perpendicular to a transverse row of cores and summing the concrete areas in-between those, it may be concluded that b_w is 10% of the corresponding dimension for a full flat plate leading to a resistive capacity in shear that is about 90% less.

3.3 According to BubbleDeck [8] and Cobiax [7]

$$V_{Rd}^{BD} = 0.6 \cdot V_{Rd,c}^{DP}, \tag{9}$$

with $V_{Rd,c}^{DP}$ is the shear capacity of the full flat plate having the same height and transverse reinforcement as Eq. (3).

The above index of “0.6” was established empirically after analysing a series of research programmes from Denmark, Germany or Netherlands [6].
M. Aldejohann [7] proposed the next equation in 2009:

$$V_{Rd,DG} = k_{DG} \cdot \frac{A_{DG}}{A_{DP}} \cdot V_{Rd,c}, \tag{10}$$

with

$$k_{DG} = 1.16 - \frac{c_o}{x} \cdot 0.03, \text{ if } 0.90 < \frac{c_0}{x} \le 1.20 \tag{11}$$

$$k_{DG} = 0.80 + \frac{c_o}{x} \cdot 0.40, \text{ if } \frac{c_0}{x} \le 0.90 \tag{12}$$

c_0 is the concrete cover for the hollow cores, x is the position of the neutral axis, A_{DG} is the area of concrete given by a 45^o degrees tilted plan intersecting the centre of a transverse row of hollow cores.

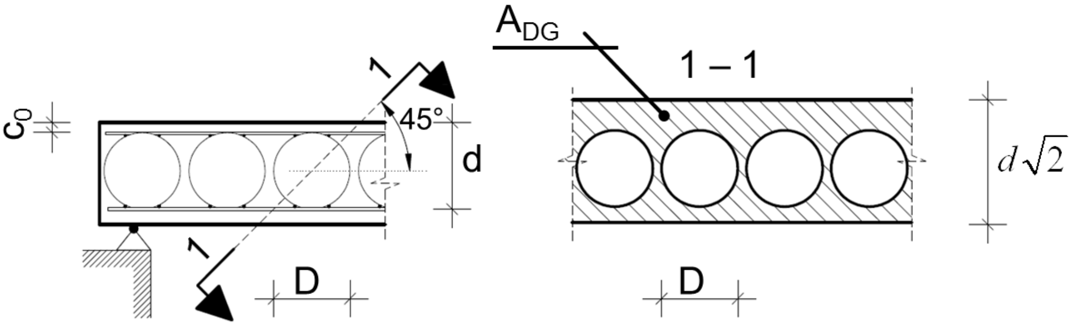

Figure 6. Establish area A_{DG} according to [7]

It may be concluded that taking into account the frequency of load levels on flat slabs, since the ratio of c_0 over x is bigger than 1.2 Eq. (10) may be applied only for increased openings and loads.

4. Conclusions

Since hollow core flat plates for rectangular structural layouts are supporting medium load levels that place the neutral axis somewhere on top of the hollow cores, giving reinforcement percentages lower that 0.5% with the exception of middle spans for higher load levels, it may be concluded that current EC2 [3] flexural design model may be successfully used in design, similar to a full flat plate.

Similar conclusions cannot be extrapolated for shear design, since both the EC2 [3] and MC 2010 [5] provide diminished values for the hollow cores flat plate as compared to the full flat plate. Eq. (10) may be used successfully instead for medium and high load levels. For low load levels additional research is mandatory to fully establish the shear behaviour.

5. References

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