Acta Technica Napocensis: Civil Engineering & Architecture Vol. 57 No. 1 (2014)
Journal homepage: http://constructii.utcluj.ro/ActaCivilEng

Inhomogeneous Behavior of Concrete Columns under Critical Loads of Design Codes

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Received 24 June 2013; Accepted 12 January 2014

Abstract

After earthquakes it becomes very visible what types of building construction have withstood the forces of the earthquake and which did not perform adequately. Analyzing the nearly collapsed and broken structures gives a good insight in the possible architectural and engineering design mistakes, faults in the detailing and the mismanagement of the construction by the building contractors. Stability calculations are critical in structural design can cause significant damage to structural column members. In discussing the stability of structural systems, the goal is the investigation of the equilibrium condition from the point view of stability and instability and determines the conditions which make the system unstable. The stability is considered for relatively thin columns with small cross section area like steel sections and is rarely discussed in usual problems of structural engineering related to reinforced concrete sections and is propounded in special structures. Concrete is a non-homogeneous and anisotropic material. Modeling the mechanical behavior of Reinforced Concrete (RC) is still one of the most difficult challenges in the field of structural engineering. There are some factors which cause the mechanical factors of concrete in right dimension are not uniform and isotropic in high columns. These factors effect on concrete elasticity modulus, Poisson coefficient and regular relations of columns critical load. There are various research works available in the literatures for determining sensitivity of modulus of elasticity to concrete strengths and other parameters but in this study, a different procedure was taken into account to investigate the effect of material uncertainty, selected seven different design codes were considered in the analyses. Instability of linear elastic columns is analyzed by the energy method. The energy method for a column provides a criterion, which determines whether the column is stable or not. Due to the numerous outputs obtained, software package have been written in Matlab and analysis on data and drawing related charts have been done.

Keywords: Concrete Columns, Elastic & Inelastic Stability, Inhomogeneous, Sensitivity Analysis

1. INTRODUCTION

Most structural failures are the result of an error made by one of the people involved in the great number of steps between the original idea and the completion of the final structure. For reinforced concrete construction, mainly inadequate column designs and over-weight structures are the cause of fatal building failure and related human victims.

Columns are structural members in buildings carrying roof and floor loads to the foundations. Most columns are termed short columns and fail when the material reaches its ultimate capacity under the

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applied loads and moments. The limit loads for columns, having major importance to a building's safety, are considered stability limits. Thus, a designer must evaluate the critical load limits. In reality, some of the design parameters in structural analysis may be disregarded which can lead to uncertainties.

Buckling, also known as structural instability may be classified into two categories: 1, bifurcation buckling and 2, limit load buckling. In bifurcation buckling, the deflection under compressive load changes from one direction to a different direction (e.g., from axial shortening to lateral deflection). The load at which the bifurcation occurs in the load-deflection space is called the critical buckling load or simply critical load. In limit load buckling, the structure attains a maximum load without any previous bifurcation, i.e., with only a single mode of deflection, [1].

The first study on elastic stability is attributed to Leonhard Euler [1707–1783], who used the theory of calculus of variations to obtain the equilibrium equation and buckling load of a compressed elastic column, (1). Most basic linear elastic problems of structural stability were solved by the end of the 19th century, although further solutions have been appearing as new structural types were being introduced. In discussing the stability of structural systems, the goal is the investigation of the equilibrium condition from the point view of stability and instability and determines the conditions which make the system unstable, [2~5].

Slender columns buckle and the additional moments caused by deflection must be taken into account in design. 1, Short columns when the ratios l_{ex}/h and l_{ey}/b are both less than 15 for braced columns and less than 10 for un-braced columns and 2, Slender columns when the ratios are larger than the values given above. The buckling load of stocky columns must be determined by taking into consideration the inelastic behavior, (Euler, 1744) [4].

$$P = \pi^2 EI \; / \; (kL)^2 \tag{1}$$

Where E is the modulus of elasticity of the column member representing the material property, I is the area moment of inertia of the cross-section, k is the column effective length factor, whose value depends on the conditions of end support of the column and L is the length of the column.

In this research, critical load or stability of inhomogeneous reinforced concrete columns have been investigated. To do this, sensitivity analysis of critical loads to various parameters such as E, I and L have been investigated. Also studying has been done on a set of concrete columns with and without inhomogeneous properties. The column section is generally square or rectangular, but circular and polygonal columns are used in special cases. Consider now columns of square cross sections. The column slenderness is defined as L/r, where $r=h/12^{0.5}$ and h=side of the square cross section =150mm. In numerical calculations, $f'c$=250 kg/cm^2 is considered. The column is reinforced symmetrically by eight axial steel bars and the steel ratio ρ=0.01 and 0.03. The cover of concrete bars is such that the axial bar centers are about 50 mm from the surface. Furthermore, Es=2e^6 kg/cm^2 and fy=4000 kg/cm^2.

2. MATERIAL PROPERTIES

2.1. Concrete Strength

Material properties affect the critical value of the buckling loads. Concrete strength is counted as one of the important parameters for the material properties in reinforced concrete structure design. The material modeling of reinforced concrete consisting generally of three phases: cement mortar, aggregate grains and reinforcing steel bars, is a strong compromise between the structural phenomena and available material parameters. In structural analysis, reinforced concrete materials are modeled as a macroscopically homogeneous material with response influences by each of the phases. Stress-strain curves are an extremely important graphical measure of a material's mechanical properties, (Fig. 1).

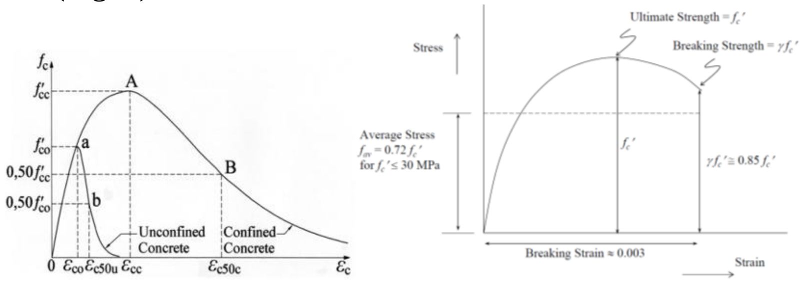


Fig. 1. Stress-strain curves, left: general, right: Todeschini

2.2. Modulus of Elasticity

Material properties can be defined through concrete strength and modulus of elasticity as proposed in different national building codes through various formulas for the same values of concrete strength (Fig. 2). Modulus of elasticity of concrete is a key factor for estimating the deformation of buildings and members, as well as a fundamental factor for determining modular ratio, *n*, which is used for the design of section of members subjected to flexure. Modulus of elasticity of concrete is frequently expressed in terms of compressive strength. In the present study, selected seven different design codes were considered in the analyses (table I). Slope of stress-strain curve is defined as elasticity modulus in concrete.

This modulus relates to the kind of concrete, concrete age and speed in loading, concrete properties and mixing percent and more importantly relates to definition of concrete elasticity modulus. According to table I, the two factors, compressive strength and weight, have relations with elasticity modulus. In concreting, by being careful about how to compact the concrete and its completion, the concrete will have much compressive strength. So all the aspects that influence in compressive strength and weight have in direct influence in elasticity modulus too.

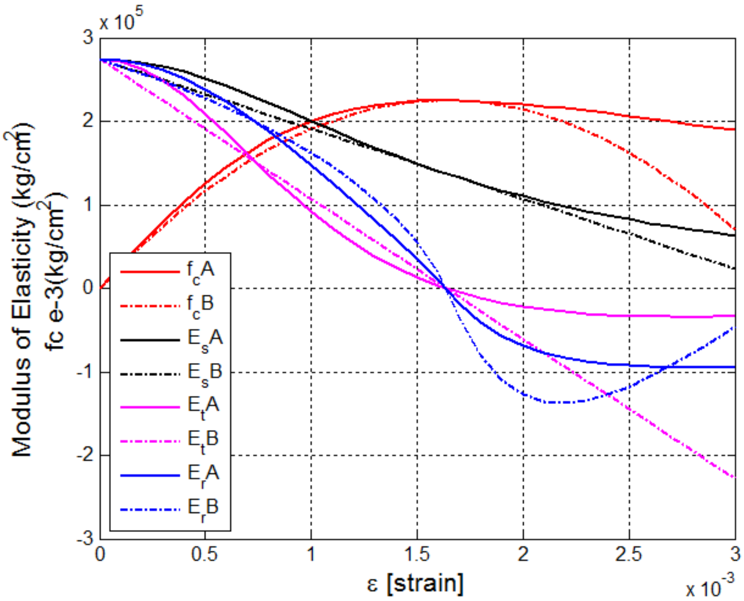


Fig. 2. A: Todeschini model, B: Hognestad model, s: secant modulus, t: tangent modulus, r: reduced modulus of elasticity

TABLE I
MATERIAL PROPERTIES AS A FUNCTION OF THE COMPRESSIVE STRENGTH

No	Code	formula	References
1	ACI-2008	$Ec = 4.73\sqrt{f_c^{'}}$ ·	American Concrete Institute
2	CEB-90	$Ec = 10\left(f_c^{'} + 8\right)^{1/3}$.	Euro-International Concrete Committee
3	TS-500	$Ec = 3.25\sqrt{f_c^{'}} + 14$ ·	Turkish Standard Committee
4	IDC-3274	$Ec = 5.7\sqrt{f_c^{'}}$ ·	Italian Design Council
5	GBJ-11-89	$Ec = \frac{10^2}{\left[2.2 + \frac{34.7}{f_c^{'}}\right]}$.	Chinese Design Council
6	ABA	$Ec = 5.0\sqrt{f_c^{'}}$ ·	Iranian Concrete Code
7	Mos-2005	$Ec = 8.3\left(f_c^{'}\right)^{0.35}$ ·	Prof. Mostofinejad, Davood [6]

2.3. Inhomogeneous Behaviour of Concrete

Using the same mixing, concrete could get different Compressive strength results in different situations. Following is some of effective factors on compressive strength of concrete. The compressive strength of concrete depends on some main factors for examples the aggregate grading, aggregate/cement ratio as well as the water/cement ratio. Also depends on some minor factors or site factors for examples grout leakage, poor compaction (the influence of gravity force on concentration of layers and type of concrete compression vibration), segregation, grading limits, poor curing and Chemical attacks such as chlorides, sulfates, carbonation, alkali-silica reaction and acids. Some studies show that humidity and good temperature in concreting after 180 days can increase the concrete strength to 3 times. In those seasons with straight sunshine, the temperature increases and humidity of concrete section decreases 2 or 3 degrees. This factor is important in column sections and surrounding beam sections of roofs.

Concrete exhibits a large number of micro-cracks, especially at the interface between coarser aggregates and mortar, even before the application of any external loads. The presence of these micro-cracks has a great effect on the mechanical behavior of concrete, since their propagation (concrete damage) during loading contributes to the nonlinear behavior at low stress levels and causes volume expansion near failure. Many of these micro-cracks are initially caused by segregation, shrinkage or thermal expansion of the mortar. Some micro-cracks may develop during loading because of the difference in stiffness between aggregates and mortar, (Mostofinejad, 2006, Tim Gudmand-Høyer and Lars Zenke Hansen, 2002), [6,7].

For example a reinforced concrete column with known high is supposed. Regarding to the column height, concreting may be done in 2 or more steps. Segregation that cause by levels and height of concreting and poor compacting of concrete may change the density of column concrete. If density of the concrete in bottom section of the column (x/L=0) equals to W_c, then the density at the top of the column section (x/L=1) would be K_w*W_c. Where K_w is the coefficient smaller than one and shows the density differences of bottom and top. For instance K_w=0.95 means the density of concrete in top of the column is 95% of density of the bottom. Sensitivity analysis of elasticity modulus to various ratios of K_w, suppose of constant compressive strength of concrete is shown in Fig. 3a.

With a change in concreting and compacting methods, the compressive strength in bottom and top of the column will be different. If the compressive strength of the concrete in bottom section of column (x/L=0) equals to f'_c, then the strength at the top of the column section (x/L=1) would be $K_c*f'_c$. Where K_c is the coefficient smaller than one and shows the compressive strength differences of bottom and top. For instance K_c=0.95 means the compressive strength of concrete in top of the column is 95% of compressive strength of the bottom. Sensitivity analysis of elasticity modulus to various ratios of K_c, suppose of constant density of concrete (k_w=1) is shown in Fig. 3b.

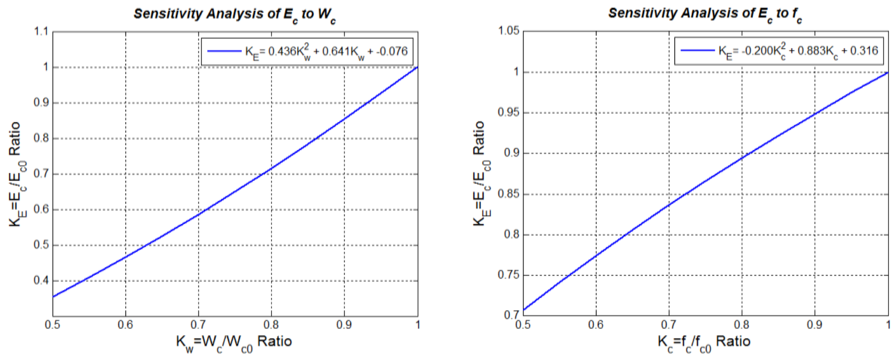


Fig. 3. Sensitivity analysis of *Ec* with *Wc* and *f'c* according to different K_w and K_c ratios

TABLE II

ELASTIC MODULUS VALUES FOR A GIVEN CONCRETE STRENGTH

References	E_c, modulus of elasticity (kg/cm³) x 10⁵			
	f'_c=250 (homogeneous)	f'_c=250 (in-homogeneous) K_c=0.9	f'_c=350 (homogeneous)	f'_c=350 (in-homogeneous) K_c=0.9
ACI-2008	2.34	2.22	2.77	2.63
CEB-90	3.16	3.08	3.45	3.58
TS-500	2.98	2.9	3.28	3.18
IDC-3274	2.82	2.68	3.34	3.17
GBJ-11-89	2.75	2.64	3.09	2.99
ABA	2.48	2.35	2.93	2.78
Mos-2005	2.53	2.44	2.84	2.74

In practice, with a change in gradation and concrete compaction, the density and the compressive strength of concrete are change (table II). It may happen that with a small change in density and without any external interference, the compressive strength of concrete decrease due to the decrease in density, it means if K_w=0.95 then K_c may equals to 0.9. Sensitivity analysis of modulus of elasticity with both K_w and K_c is shown in Fig. 4. If the density of concrete in top of the column is %95 of the density in bottom and it may cause that the compressive strength in top of the column be %90 of the bottom, thus the elasticity modulus of concrete in top of the column become %88 of the elasticity in bottom, (Fig. 5).

In this form the elasticity follows a second order equation (K_E). This second order equation shows the inhomogeneous behavior of concrete in column height and one can calculate the critical load. These second order equation is used to estimate the changes of elasticity modulus, (Vahid Shahsavar, 2010 [8].

$$E_x = \left(0.003\xi^2 - 0.125\xi + 1\right)E_0 \qquad (2)$$

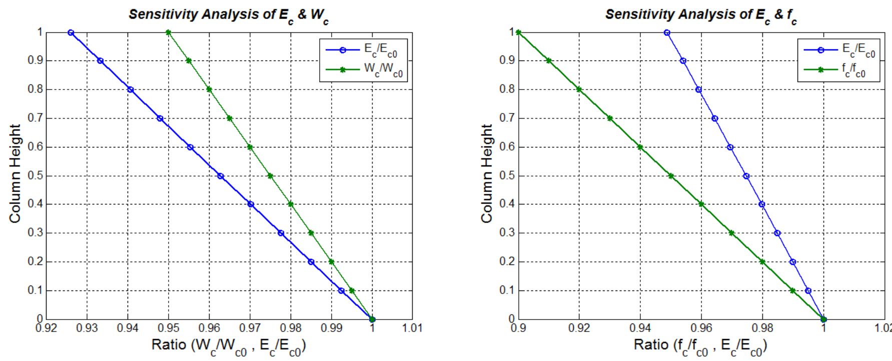


Fig. 4. Sensitivity analysis of E_c with W_c or f'_c according to column nonparametric height left: K_w=0.95, right: K_c=0.90)

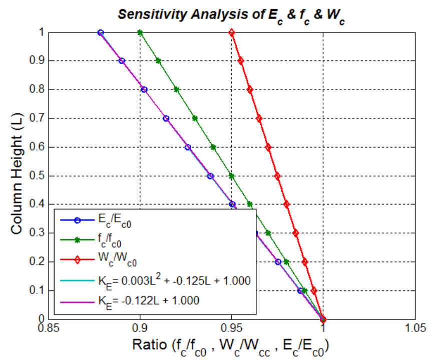


Fig. 5. Sensitivity analysis of E_c with W_c and f'_c according to column nonparametric height (K_w=0.95 & K_c=0.90)

3. STABILITY PROBLEM

3.1. Eigenvalue and stability of Column

Instability of linear elastic columns is analyzed by the energy method. The energy method for a column provides a criterion, which determines whether the column is stable or not.

$$v = \delta.\sin\left(\frac{n.\pi.x}{L}\right) \tag{3}$$

Where v is shape function and,

$$\xi = \frac{x}{l}, dx = L.d\xi \tag{4}$$

$$D_w = \frac{1}{2}P\int_0^L \left(\frac{d_v}{d_x}\right)^2 d_x \tag{5}$$

$$D_u = \frac{1}{2}\int_0^L E_x I_x \left(\frac{d_v^2}{d_x^2}\right)^2 d_x \tag{6}$$

$$D_w = \frac{1}{2}P\int_0^L \left(\frac{n\pi\delta}{L}\cos(n\pi\xi)\right)^2 L.d_\xi = \frac{n\pi^2\delta^2(\sin(2n\pi)+2n\pi)}{4L} \tag{7}$$

$$D_u = \frac{1}{2}\int_0^L \left(0.003\xi^2 - 0.125\xi + 1\right)\times E_0 I_0)\left(-\frac{(n\pi)^2}{L^2}\delta\sin(n\pi\xi)\right)^2 L.d_\xi \tag{8}$$

$$= \frac{n\pi\delta^2 E_0 I_0\left(3\sin(2n\pi) + 4n^3\pi^3 - 6n^2\pi^2\sin(2n\pi) - 6n\pi\cos(2n\pi)\right)}{8000L}. \tag{9}$$

$$n = 2, 2^{nd}\ mood : P_{cre} = \frac{\pi^2 E_0 I_0}{2000L^2}\left(8L^2 + 8000 - 3L^2/\pi^2 - 500L\right) \tag{10}$$

To compare the critical load for liner (first order) equation of elasticity modulus is also estimated.

$$E_x = (-0.122\xi + 0.999)E_0 \tag{11}$$

$$n = 1, 1^{st}\ mood : P_{cre} = \frac{\pi^2 E_0 I_0}{1000L^2}(61L - 999) \tag{12}$$

$$n = 2, 2^{nd}\ mood : P_{cre} = \frac{\pi^2 E_0 I_0}{250L^2}(61L - 999) \tag{13}$$

Supposing a liner relation of elasticity modulus of concrete in column height instead of second order equation, critical load of first mood of buckling has %0.032 error and the error of second buckling mood is %0.044. Column critical load by homogeneous behave of concrete with unique length of column will be 9.869 *EI*. It means that if changing of the elasticity modulus equals to K_E=0.88 then the critical load decrease from 9.869 to 9.261, around %94.

3.2. *Sensitivity of Critical Buckling Load on Material Properties*

There are various research works available in the literatures for determining sensitivity of modulus of elasticity to concrete strengths. In this study, a procedure was taken into account to investigate the effect of material uncertainty. In the present study, selected seven different design codes were considered in the analyses (table I). Relationships of f'_c and E_c are expressed in MPa and in GPa, respectively. Relationship curves of elasticity Modulus and various concrete strengths for different design codes are shown in Figs. 6 and 7, (Korkmaz *et al*., 2011, Mostofinejad, 2006), [5,6].

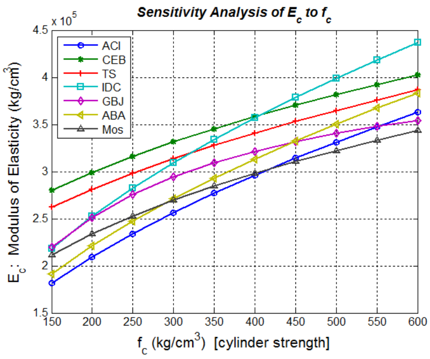


Fig. 6. Relationship curves of elasticity Modulus and various concrete strengths for different design codes, K_c=1

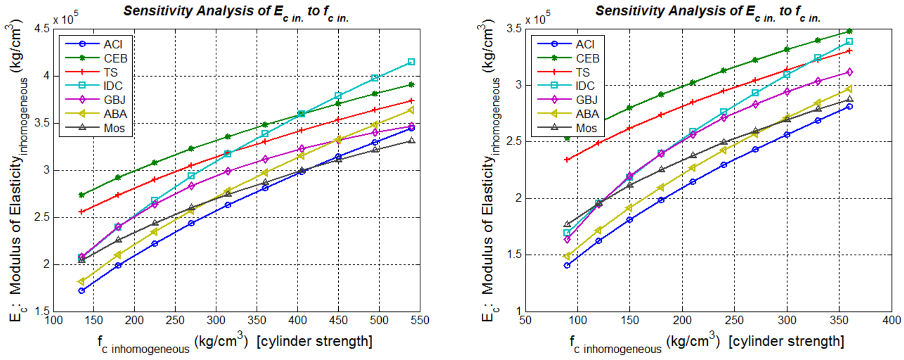


Fig. 7. Relationship curves of elasticity modulus and various concrete strengths for different design codes, left: K_c=0.9, right: K_c=0.6 [K_c=f'_c/f'_{c0}]

3.3. *Tangent-Modulus Theory*

According to the tangent-modulus theory (Engesser theory) of inelastic buckling, column remains straight until inelastic critical load is reached. At that value of load, the column may undergo a small lateral deflection. The resulting bending stresses are superimposed upon the axial compressive stresses S_A. Since the column starts bending from a straight position, the initial bending stresses represent only a small increment of stress. Therefore, the relationship between the bending stresses and the resulting strains is given by the tangent modulus. Since the tangent modulus E_t varies with the compressive stress $S_{P/A}$, we usually obtain the tangent-modulus load by an iterative procedure.

$$\sigma_T = \frac{2f_c''\left(\varepsilon_c/\varepsilon_0\right)}{1+\left(\varepsilon_c/\varepsilon_0\right)^2} \tag{14}$$

$$f_c'' = K_s f_c' \tag{15}$$

$$\varepsilon_c = \frac{\varepsilon_0 f_c'' - \varepsilon_0\left(f_c''^2 - \sigma_T^2\right)^{0.5}}{\sigma_T} \tag{16}$$

$$\varepsilon_0 = \frac{1.71 f_c''}{E_{c0}} \tag{17}$$

$$E_s = \frac{f_c'' + \sqrt{f_c''^2 - \sigma_T^2}}{\varepsilon_0} \tag{18}$$

$$E_t = E_s\sqrt{1-\frac{\sigma^2}{f_c''^2}} \tag{19}$$

Where f_c is the compressive stress, f_{ck} is the characteristic compressive strength of cubes, ε_c is the compressive strain, ε_0 is the strain corresponding to f_{ck} = 0.002, ε_{cu} is the ultimate compressive strain = 0.003 (Based on Todeschini concrete stress-strain equation).

3.4. Reduced-Modulus Theory

The tangent-modulus theory is distinguished by its simplicity and ease of use. However, it is conceptually deficient because it does not account for the complete behavior of the column. The results of such analyses show that the column bends as though the material had a modulus of elasticity between the values of E and E_t. This "effective modulus" is known as the reduced modulus E_r , and its value depends not only upon the magnitude of the stress (because E_t depends upon the magnitude of the stress) but also upon the shape of the cross section of the column. Thus, the reduced modulus E_r is more difficult to determine than is the tangent modulus E_t. In the case of a column having a rectangular cross section, the equation for the reduced modulus is:

$$E_r = \frac{4EE_t}{(\sqrt{E}+\sqrt{E_t})^2} \tag{20}$$

$$P_r = \frac{\pi^2 E_r I}{L^2} \tag{21}$$

$$\sigma_r = \frac{\pi^2 E_r}{\left(\frac{L}{r}\right)^2} \tag{22}$$

3.5. Effects of Confinement on Interaction Diagrams

The effects of confinement on a structural column in a building are mainly due to the presence of lateral reinforcement provided over the column height. It results in higher capacity and ductility of a column that help to prevent the column from brittle failure. Several stress-strain relationships of confined concrete available in literature such as Kent-Park, Sheikh-Uzumeri, Mander *et al.*, Yong-Nawy, Cusson- Paultre, Diniz-Frangopol, Kappos- Konstantinidis, Hong-Han, and Kusuma-Tavio. Kent-Park and Mander et al. relationships are adopted in the study. Also the unconfined concrete model adopted in the study is Todeschini stress-strain model, (Tavio, 2008), [9].

3.6. Elastic Second-Order Analysis

In defining the critical load, the main problem is the choice of stiffness *EI* that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve. Second-order analysis shall consider material nonlinearity, member curvature and lateral drift, duration of loads, shrinkage and creep, and interaction with the supporting foundation. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with results of comprehensive tests of columns in statically indeterminate reinforced concrete structures. Elastic second-order analysis shall consider section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and the effects of load duration. The stiffness (*EI)* used in an analysis for strength design should represent the stiffness of the members immediately prior to failure. This is particularly true for a second-order analysis that should predict the lateral deflections at loads approaching ultimate, (Tavio, 2009), [10]. The moments of inertia of compression and flexural members, *I*, shall be permitted to be computed as follows (ACI-318):

$$I_m = \left(0.80 + 25\frac{A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - 0.5\frac{P_u}{P_0}\right) I_g \le 0.875 I_g \tag{23}$$

Where A_{st} is the rebar's area, A_g is the gross section area, P_u and M_u are the factored loads and P_0 is the critical buckling load for the column. P_u and M_u shall be determined from the particular load combination under consideration or the combination of P_u and M_u determined in the smallest value of *I*. *I* need not be taken less than $0.35I_g$.
When sustained lateral loads are present, *I* for compression members shall be divided by $(1 + \beta_d)$. The term β_d is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination, but shall not be taken greater than 1.0.
In lieu of a more precise analysis *EI* may be taken either as:

$$EI_{mm} = \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d} \tag{24}$$

$$I_s = C_t \gamma^2 \rho_t I_g \tag{25}$$

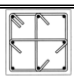
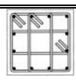
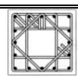
$$\gamma = \frac{d}{h} \tag{26}$$

$$\rho_t = \frac{A_{st}}{A_g} \tag{27}$$

Where C_t introduced in table III. The greatest emphasis is currently being placed on the analysis of instabilities and bifurcations caused by propagation of softening damage or fracture in materials, which is important not only from the physical and engineering viewpoint, but also from the viewpoint of computational modeling. Fig. 8 shows cracked moments of inertia of compression and flexural members according to different reinforcement ratios. Also fig. 9 shows variety of EI_{mm} values according to different reinforcement ratios.

TABLE III

DEFINITION OF C_T BASED ON CROSS SECTION SHAPE AND NUMBER OF BARS

Shape			
Number of bars	8	12	16
C_t	2.2	2.1	2.06

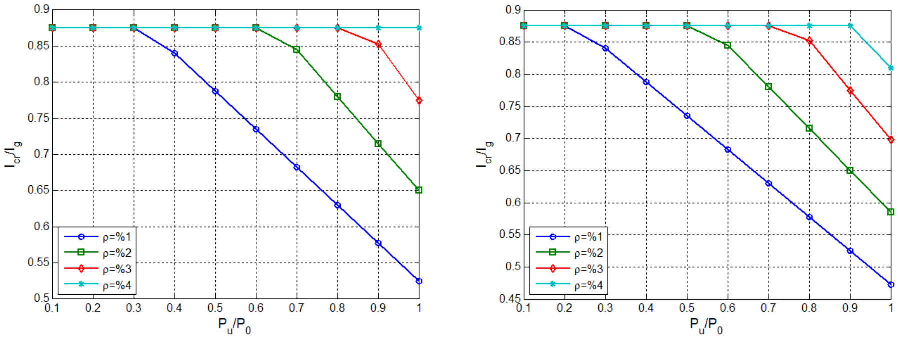


Fig. 8. Cracked moments of inertia of compression and flexural members according to different reinforcement ratios, left e=0, M_u=0, right e=%5, M_u=0.05*h*P_u

The deformations due to shear forces are neglected in the classical ending theory, since the cross sections are assumed to remain normal to the deflected beam axis. This assumption is usually adequate; however, there are some special cases when it is not. The shear deformations can be taken into account in a generalization of the classical bending theory called Timoshenko beam theory. Usually elastic modulus changes with critical loads, fig 10.

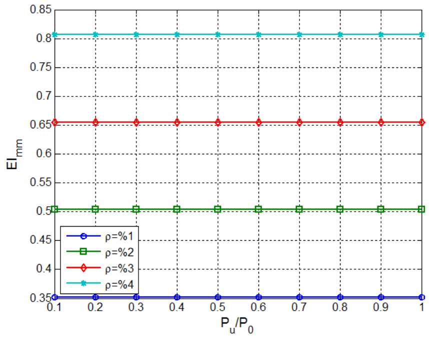


Fig. 9. EI_{mm} according to different reinforcement ratios *(β_d=0, C_t=2.2, ACI318)*

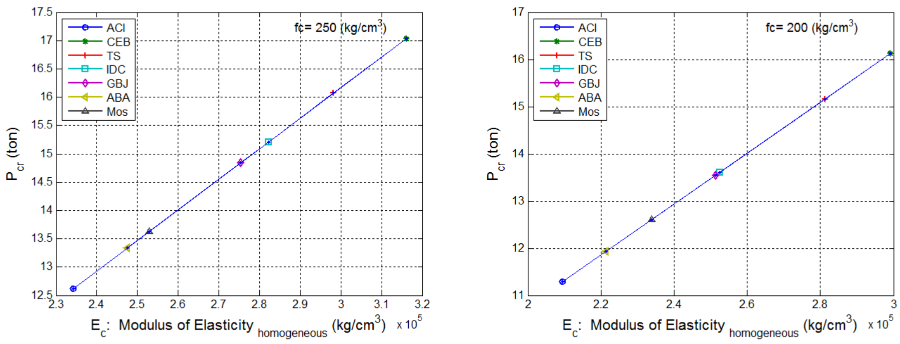


Fig. 10. Elastic modulus change with critical load according to different codes, (left K_w=1, right K_w=0.8)

When the column is stocky, or built up (latticed or battened) or of a composite-type construction, the application of Euler (classical) beam theory will overestimate the buckling loads. This is due to the neglect of transverse shear deformation in the Euler beam theory. A more refined beam theory, known as the first-order shear deformation theory or Timoshenko beam theory, that incorporates the shear deformation effect was proposed by Engesser (1891) and Timoshenko (1921). According to the Engesser–Timoshenko beam theory, the stress–resultant–displacement relations are given by:

$$M = E.I.\frac{d\theta}{d\bar{x}} \tag{28}$$

$$V = K_s GA\left(\theta + \frac{d\bar{w}}{d\bar{x}}\right) \tag{29}$$

In which $\bar{x}$ s the longitudinal coordinate measured from the column base, M the bending moment, V the transverse shear force, θ the rotation in the Engesser–Timoshenko column and w the transverse deflection. The shear correction coefficient K_s in the (29) is introduced to account for the difference in the constant state of shear stress in the Engesser–Timoshenko column theory and the parabolic variation of the actual shear stress through the depth of the cross-section, (Bazant, 2000, Timoshenko, 1921), [1,11].

$$\frac{d^4 w}{dx^4} + \bar{k}\frac{d^2 w}{dx^2} = 0 \tag{30}$$

Where $x = \frac{\bar{x}}{L} \qquad w = \bar{w}$ (31)

$$w = C_1 \sin\sqrt{\bar{k}}x + C_2 \cos\sqrt{\bar{k}}x + C_3 x + C_4 \tag{32}$$

For Pinned-Pinned end: $\bar{w} = 0$ and $dv\big/d\bar{x} = 0$ then $\sin\sqrt{\bar{k}} = 0$ (33)

By comparing the stability criteria of the Timoshenko columns with their Euler counterparts in (1), it is clear that the Timoshenko critical load P^T_{cr} and the Euler critical load P^E_{cr} are related by:

$$P^T_{cr} = \frac{P^E_{cr}}{1 + \frac{P^E_{cr}}{K_s GA}} \tag{34}$$

The higher-order shear deformation beam theory, proposed by Bickford (1982) and Heyliger and Reddy (1988), does away with the need of the shear correction factor by assuming that the transverse normal to the centroid axis deforms into a cubic curve. Using this Bickford–Reddy beam theory, Wang *et al*. (2000) showed that for pinned ended columns, fixed ended columns and elastic rotationally restrained ended columns, the Bickford–Reddy critical load P^R_{cr} is related to the Euler critical load P^E_{cr} by:

$$P^R_{cr} = P^E_{cr}\left(\frac{1 + \frac{P^E_{cr}\alpha^2 \bar{D}_{xx}}{GAD_{xx}}}{1 + \frac{P^E_{cr}\bar{D}_{xx}}{GAD_{xx}}}\right) \tag{35}$$

Where $\alpha = \frac{4}{3h^2}, \bar{D}_{xx} = D_{xx} - 2\alpha F_{xx} + \alpha^2 H_{xx}$ and $(D_{xx}, F_{xx}, H_{xx}) = \int_A (z^2, z^4, z^6) EdA$ are the higher-order rigidities and h is the height of the column cross section. For example, for a square cross-section column, (35) simplifies to:

$$P^R_{cr} = P^E_{cr}\left(\frac{1 + \frac{P^E_{cr}}{70GA}}{1 + \frac{17P^E_{cr}}{14GA}}\right) \tag{36}$$

Fig. 11 shows Critical Load change with δ/L and L/r ratios according to different codes for three types of theories, Euler, Timoshenko and Reddy. Also comparisons of the results of the theories are represented in table IV.

TABLE IV

CRITICAL LOADS VALUES FOR VARIOUS REFERENCES

References	P_{cr}, (ton)	
	Euler (equation) K_w=1	Timoshenko (equation) K_w=1
ACI-2008	39	38.92
CEB-90	52.65	52.53
TS-500	49.67	49.56
IDC-3274	47	46.9
GBJ-11-89	45.86	45.76
ABA	41.23	41.14
Mos-2005	42.11	42.02

4. INHOMOGENEOUS FORMULATION OF COLUMN BY THE ENERGY METHOD

Euler's equation is given from solving of a differentiating of deformation curves and in some parts it is a little complex and we should use approximation equations based on system energy. Elasticity modulus, *E*, is defined by ACI-318 as:

$$E_0 = 0.1347 W_c^{1.5}\sqrt{f_c^{'}}\left(\frac{kg}{cm^2} - ACI\;318\right) \tag{37}$$

$$W_{cx} = \left(1-\xi\left(1-K_w\right)\right)W_{c0} \tag{38}$$

Where W_c and W_{c0} are the density of concrete and W_{cx} is the inhomogeneous formulation of the density. Inhomogeneous formulations of the compression strength, $f^{'}_{cx}$, and elasticity modulus, E_x, are represented as:

$$f_{cx}^{'} = \left(1-\xi\left(1-K_w\right)\right)f_{c0}^{'} \tag{39}$$

$$E_x = \left(1-\xi\left(1-K_w\right)\right)^{1.5}\sqrt{\left(1-\xi\left(1-K_w\right)\right)}\times E_0 \tag{40}$$

$$\alpha = 1-K_w \tag{41}$$

$$E_x = \left(1-\alpha\xi\right)^{1.5}\sqrt{\left(1-\alpha\xi\right)}\times E_0 = \left(1-\alpha\xi\right)^2 E_0 \tag{42}$$

A Pined-pined column section with *P* load should be analyzed. In virtual work theorem (spiritual form of original shape), virtual work, *W*, equals with strain energy, *U*, (5, 6). The actuate form of section is supposed as the first mood and an element arch length of column is shown by δ or d_s, (3). To calculate accomplished work with external load *P*, we have *du=ds-dx* that equals to displacement of vertical load, (Vahid Shahsavar, 2011, Krauberger N., 2012), [12,13].

Strain energy of column that relates to flexure displacement, compressive load and shear force could be calculated by omission of shear force and for a pined-pined column section and with (3~6) we can follow the below:

$$D_w = \tfrac{1}{2}P\int_0^L\left(\frac{\pi\delta}{L}\cos\left(\pi\xi\right)\right)^2 L.d_\xi = \frac{\pi^2\delta^2}{2L} \tag{43}$$

$$D_u = \tfrac{1}{2}\int_0^L\left(1-\alpha\xi\right)^2 E_0 I_0\left(-\frac{\pi^2}{L^2}\delta\sin\left(\pi\xi\right)\right)^2 L.d_\xi$$

$$= \frac{\pi^2\delta^2 E_0 I_0\left(\alpha^2/3 - \alpha^2/2\pi^2 - \alpha + 1\right)}{4L^3}. \qquad = \frac{\pi^2\delta^2 E_0 I_0\left(\alpha^2/3 - \alpha^2/2\pi^2 - \alpha + 1\right)}{4L^3} \tag{44}$$

$$D_w = D_u \rightarrow P_{cre} = \frac{\pi^2 E_0 I_0}{L^2}\left(\alpha^2/3 - \alpha^2/2\pi^2 - \alpha + 1\right) \tag{45}$$

$$P_{cre}/P_{cr0} = \left(\alpha^2/3 - \alpha^2/2\pi^2 - \alpha + 1\right) \tag{46}$$

To create the stress-slenderness sensitivity curves related to the various codes and various inhomogeneous parameters (K_w & K_c), the results of the stability analysis were inspected and collected and the outcome is indicated according to the related figures. The sensitivity curves were obtained in the different levels of slenderness and various design codes, based on the four famous modulus theories, as follows: 1- *Euler*, 2- *Tangent*, 3- *Secant*, 4- *Reduced* and the results were compared.

In fig. 12, buckling stress versus slenderness for various types of modulus theories and ACI318 code has been demonstrated, (Left: K_w=0.9, right Kw=1.0). In figs. 13 to 17, buckling stress versus slenderness for various inhomogeneous parameters (K_w & K_c) and various design codes have been demonstrated. Other studied curves and relationships can't be figured in this paper because of some locative limitations.

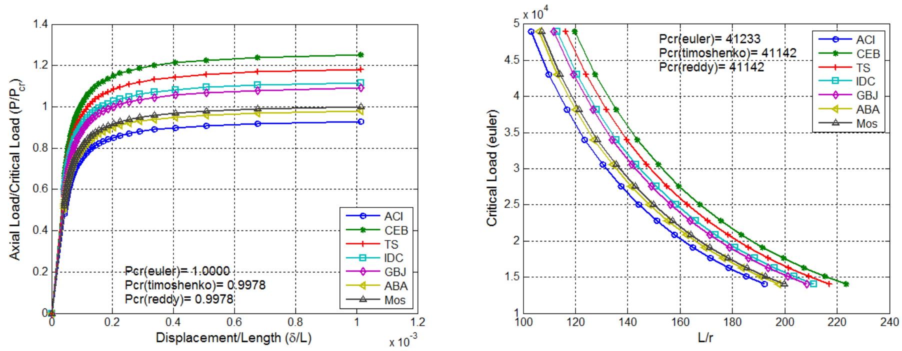


Fig. 11. Critical Load change with δ/L and L/r ratios according to different codes

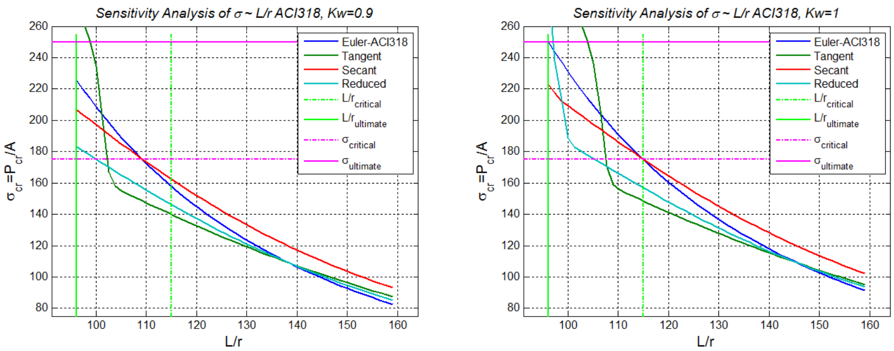


Fig. 12. Buckling stress versus slenderness for various types of modulus theories and ACI-318 code. I=1.0 I_g, left: K_w=0.9, right K_w=1.0

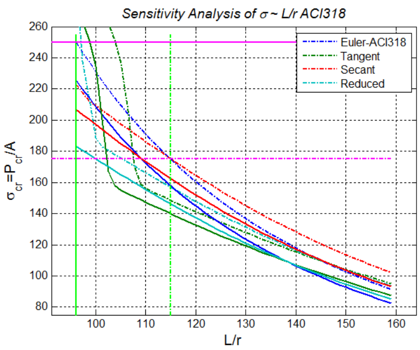


Fig. 13. Buckling stress versus slenderness for various types of modulus theories and ACI-318 code. I=1.0 I_g, (-)K_w=0.9, (-.) K_w=1.0

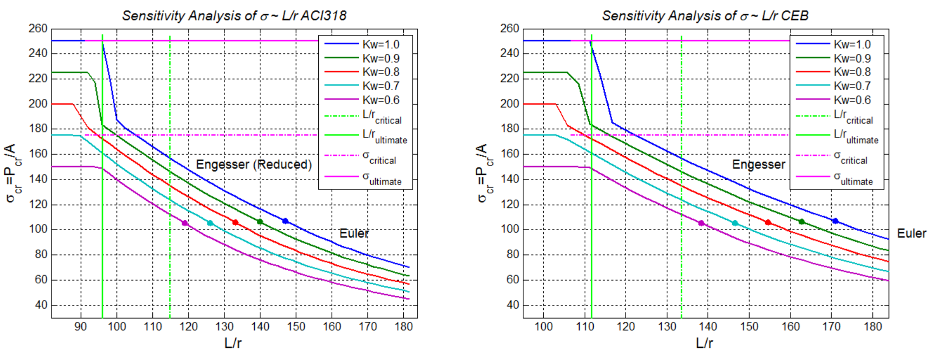


Fig. 14. P_u/P_0=1, ρ=%2, e=0, I_m =0.65I_g, left ACI code, right CEB code. K_w=0.6 to 1

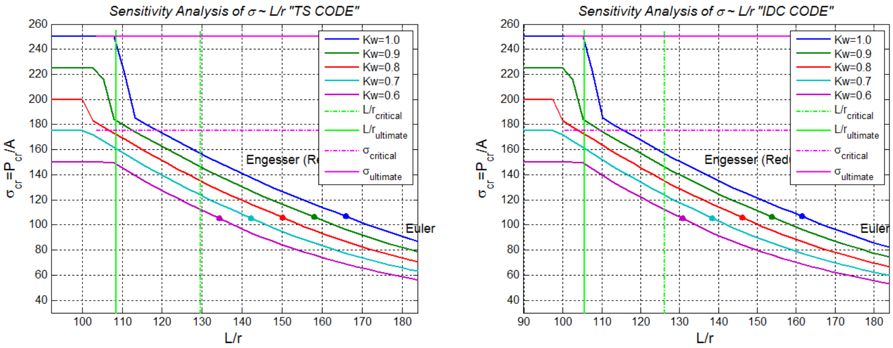


Fig. 15. P_u/P_0=1, ρ=%2, e=0, I_m =0.65I_g, left TS code, right IDC code. K_w=0.6 to 1

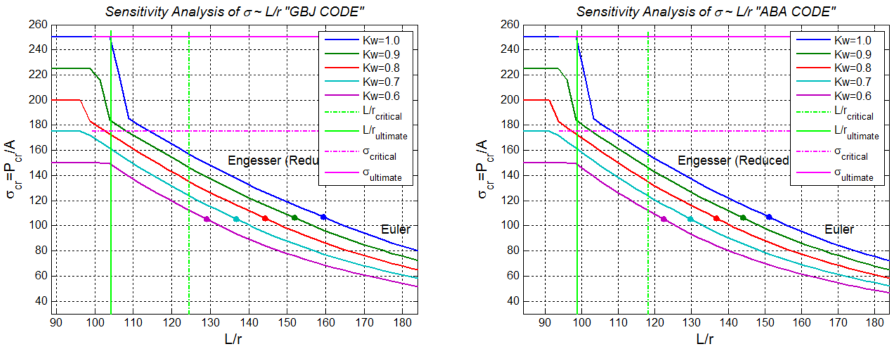


Fig. 16. P_u/P_0=1, ρ=%2, e=0, I_m =0.65I_g, left GBJ code, right ABA code. K_w=0.6 to 1

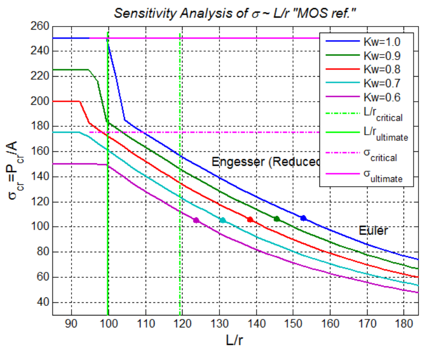


Fig. 17. P_u/P_0=1, ρ=%2, e=0, I_m =0.65I_g, MOS reference. K_w=0.6 to 1

5. CONCLUSION

In this study, the effect of material uncertainty on the buckling of inhomogeneous reinforced concrete columns was investigated. Material properties affect the critical value of the buckling loads. Material uncertainty was represented by the main important parameters of concrete as concrete strength and elastic modulus. Using the same mixing, concrete could get different Compressive strength results in different situations. In practice, with a change in gradation and concrete compaction, the density and the compressive strength of concrete are change. Also sensitivity analysis of critical loads to various parameters such as E, I and L was investigated. Selected seven different design codes were considered in the analyses. Based on the hyper-geometric solution, numerical values of the buckling capacities for inhomogeneous reinforced concrete columns are computed and presented.

Results show if the density of concrete in top of the column is %95 of the density in bottom and it may cause that the compressive strength in top of the column be %90 of the bottom, thus the elasticity modulus of concrete in top of the column become %88 of the elasticity in bottom. In this form the elasticity follows a second order equation.

If changing of the elasticity modulus equals to K_E=0.88 then the critical load decrease around %94. Based on inhomogeneous behavior of concrete in column height and for compressive strengths of concrete smaller than 400, ACI code obtains smallest and CEB code obtains largest value of elasticity modulus, also critical load of ACI has smallest and critical load of CEB has largest values. For compressive strengths of concrete larger than 400, place of ACI replaced with MOS and CEB replaced with IDC. The ranking of Design codes in presenting of elasticity modulus changes with increasing of compressive strengths of concrete. Critical load versus elasticity modulus point of all design codes set on straight line. For specific amount of column slenderness, critical loads outcome from various codes may differ up to 35%. Critical loads outcome from ACI and ABA codes are conservative and from GBJ, IDC, TS and CEB codes are respectively non-conservative. All buckling stress-slenderness curves for various inhomogeneous parameters are parallel.

6. REFERENCES

[1] Bazant, ZdeneÏk P., “Structural stability”, International Journal of Solids and Structures, no 37, 2000.

[2] Duan W. H. and Wang C. M., “Exact solution for buckling of columns Including self-weight”, Journal of Engineering Mechanics © ASCE, 2008.

[3] Elishakoff, Isaac, “Eigenvalues of inhomogeneous structures”, CRC Press, 2005.

[4] Euler, L., “Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes (appendix, de curvis elasticis)”, marcum michaelem bousquet, Lausanne (Sec. 1.2), 1744.

[5] Korkmaz, Kasim et al., “Uncertainty modeling of critical column buckling for reinforced concrete buildings”, Sadhana, Indian Academy of Sciences, Vol. 36, Part 2, 2011.

[6] Mostofinejad, Davood, “Reinforced concrete structures”, Arkaneh danesh, Isfahan university, first edition, 2006.

[7] Tim Gudmand-Høyer & Lars Zenke Hansen, “Stability of concrete columns”, PhD thesis, Technical University of Denmark, 2002.

[8] L. Shahsavar, Vahid, “Stability of non-uniform orthotropic inhomogeneous rectangular plates”, Theoretical and Applied Mechanics, AZUAC, Azerbaijan University of Architecture and Construction, Vol. 17, No 1, 2010.

[9] Tavio, “Spline nonparametric regression analysis of stress-strain curve of confined concrete”, Civil Engineering Dimension, Vol. 10, No. 1, 2008.

[10] Tavio, T. et al., “Effects of confinement on interaction diagrams of square reinforced concrete columns”, Civil Engineering Dimension, Vol. 11, No. 2, 2009.

[11] Timoshenko, S. P., "On the correction for shear of the differential equation of transverse vibrations of prismatic bars," Philos. Mag. [Ser. 6], 21: 747 (Sec. 1.7), 1921.

[12] L. Shahsavar, Vahid, “Forced vibration of non-uniform orthotropic inhomogeneous rectangular plates clamped in two sides”, Theoretical and Applied Mechanics, AZUAC, Azerbaijan University Of Architecture and Construction, Vol. 21, No 1, 2011.

[13] Krauberger N., S. Bratina, M. Saje, S. Schnabl and I. Planinc, “Inelastic buckling load of a locally weakened reinforced concrete column”, Engineering Structures, vol. 34, 278–288, 2012.