An Alternative Method For The Determination of Inelastic Displacements Using Pushover Analysis and Directly Generated Inelastic Spectra

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Abstract

The present project aims to present an alternative method for the determination of the inelastic displacements using pushover analysis and directly generated inelastic spectra. The determination of the target displacement is similar to Fajfar`s N2 method, employed in Eurocode 8. Unlike the EC8, where empirical formulas are used for describing the inelastic seismic demand, the proposed method uses directly generated constant-ductility inelastic response spectra, trying to reproduce actual nonlinear structural response by means of a chosen hysteretic representation of the system. Using the inelastic response history of Single Degree of Freedom (SDOF) systems makes possible to eliminate uncertainties regarding the determination of the inelastic spectra. The proposed method represents an alternative for nonlinear time-history analysis by using a specific demand, in terms of recorded earthquake ground motion. In the case of elastic design spectra compatible with a specific accelerogram, the results are comparable to those obtained by nonlinear pushover analysis. In order to avoid errors, both pushover analysis and incremental dynamic analysis were conducted with the same software package, on the same computational model.

Rezumat

Lucrarea prezintă o metodă pentru determinarea deplasărilor inelastice prin metoda pushover şi spectre inelastice generate direct. Determinarea deplasării ţintă este într-un fel similară cu metoda N2 a lui Fajfar, care a fost inclusă şi în Eurocode 8. În încercarea de a reproduce răspunsul real al structurii şi spre deosebire de metoda din codul european spectrele inelastice sunt determinate în mod direct. Folosirea analizei dinamice neliniare pentru oscilatori cu un grad de libertate face posibilă eliminarea unor incertitudini în ceea ce priveşte răspunsul inelastic al structurii. Din cauza faptului că pentru datele de intrare sunt utilizate accelerograme, metoda reprezintă o alternativă pentru analiza time-history. În cazul în care există accelerograme compatibile cu înregistrarea seismică, rezultatele pot fi comparate cu cele ale analizei statice neliniare. Pentru a înlătura erorile cauzate de modelele distincte de calcul, atât analizele pushover cât şi cele time-history au fost efectuate cu acelaşi pachet software, pe acelaşi model de calcul.

1. Introduction. Pushover analysis in a nutshell

Through design code inclusion (ATC-40 [1], FEMA 356 [2], 440 [3] and EC8 [4]) pushover

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analysis has become latterly a widely used nonlinear analysis method. The determination of inelastic displacements has been made possible thereby with sufficient accuracy and without using time-expensive analysis methods. Generally speaking, all aspects of pushover analysis can be divided into those referring, in the one hand, to structure capacity and, on the other hand, to seismic demand. Capacity is represented by the force-displacement curve, developed by using a nonlinear incremental analysis. The invariable or adaptive lateral force distribution stands for the height-wise distribution of structural stiffness. In order exclude the necessity of carrying out a nonlinear response history analysis for the determination of the inelastic demand of SDOF systems, empirical relationships have been introduced, which serve for the overdamping of the elastic spectra (Capacity Spectrum Method (CSM) ATC-40[1] case), or the determination of the constant ductility inelastic spectra (N2 Method - EC8 case) [4] [5]. The second method is widely used in Europe due to its inclusion in the Eurocode 8. According to the N2 method, the inelastic spectra is developed through applying empirical Rμ-μ-T relationships, like those of Miranda and Bertero [6], Newmark and Hall [7], Krawinkler and Nassar [8], Fajfar, Vidic and Fischinger [9]. Note that, the aforementioned relationships can be exclusively applied to design (smooth) elastic spectra. The limitations of the pushover analysis are also well-known: the incapability to account for progressive distribution of the strength deterioration and the influence of higher mode effects. The main reason for these limitations has been identified to be the use of invariant lateral mode-distribution. [10]. Mention must be made, that the newly developed adaptive load distribution didn't produce the expected results, at the same time the conceptual simplicity of the invariant load distribution has been lost. [11, 12]

2. A pushover analysis method using directly generated inelastic spectra

The use of constant ductility inelastic spectra for CSM has been studied by Bertero [13], Reinhorn [14], more recently Chopra [15], and then Fajfar [5] proposed the use of inelastic spectra instead of the overdamped elastic spectra, within the confines of CSM. Being applied to smoothed elastic design spectra whereon the application of the Rμ-μ-T relationships is facile, the basic variant of the N2 method cannot be applied to one specific earthquake record, represented by the accelerogram. Nevertheless, Fajfar [5] indicates the application "of a specific acceleration time history (…) which takes into account specific hysteretic behaviour" as a possible extension of the method. Notwithstanding, this may seem contradictory to the static feature of the CSM method. Genkturk and Elnashai [16] suggested the use of inelastic response history analysis of SDOF systems. They mention that the latter" is a matter of fractions of a second on an average personal computer " and " in addition, it eliminates approximations and hence the errors introduced into the solution with the use of equivalent linear systems" [16]. Based on this principle, we propose hereinafter an alternative method, which - likewise that of Genkturk and Elnashai[16] - uses inelastic response history analysis for the determination of the inelastic seismic demand, namely the constant ductility inelastic spectra. The concept of direct determination of the inelastic spectra through nonlinear time-history analysis has been used also by Aschheim in the Yield Point Spectra (YPS) [22] method. However, there are some differences between the proposed and the YPS method, as follows:

-in the proposed method, the ductility demand is determined directly through the DTHA of the SDOF oscillator by drawing the μ-T diagram, characterized by constant strength reduction R_μ (fig. 2a), while YPS requires further numerical approximations after plotting the yield coefficient-yield displacement-period (C_y-d_y-T) diagram.

- the proposed method retains the spectral acceleration-spectral (ultimate) displacement ADRS format, while YPS uses the coefficient-yield displacement representation (C_y-d_y)

The proposed method is similar to the N2 method, with the distinction that it is applied to a specific earthquake and the deduction of the inelastic spectra is based on the actual behaviour of the equivalent SDOF oscillator. Due to space limitations, the complete procedure cannot be presented

here. For further issues regarding capacity (determination of the equivalent SDOF capacity curve, bilinear idealization), the reader is referred to [4, 5]. The demand-related issues of the proposed method will be treated hereinafter.

Unlike the basic variants of the method [4, 5], the seismic demand is represented by constant ductility inelastic spectra developed from a specific earthquake record. First, the elastic demand spectra are generated in terms of acceleration-displacement (Sa-Sd).
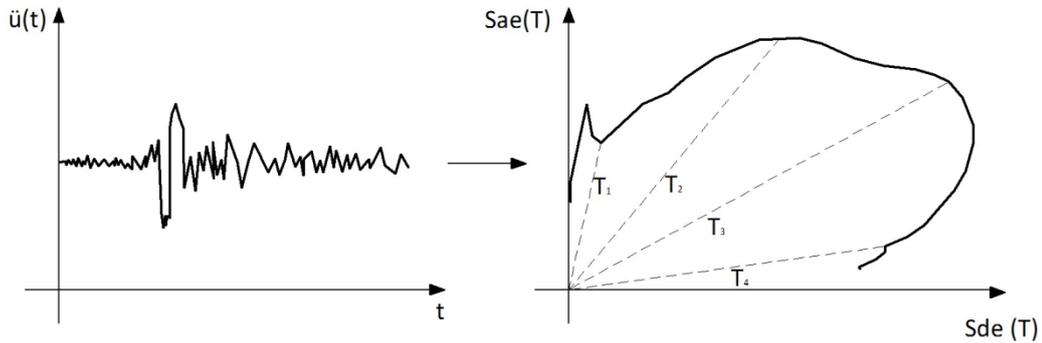
Figure 1 .The development of the elastic demand spectra from the earthquake recording

As it can be seen on Fig. 1, the acceleration and the displacements are plotted one against the other. The periods are represented by radial lines. Then the SDOF oscillators with predetermined period spectra and assumed hysteretic behaviour are subjected to inelastic response history analyses to develop constant ductility inelastic spectra.

Practically speaking, the reduction factor R_{μ} can be calculated by dividing the accelerations corresponding to the inelastic and elastic systems (Eq. 1). Finally, using the inelastic response history analysis tool, a μ-T diagram is developed (fig. 2a). The ductility μ at the period T^* characterizes the seismic demand, and the inelastic spectra have to be developed at this constant ductility (fig.2b.). The latter aspect is considered to be the major advantage of the method, because it allows the direct determination of ductility demand μ corresponding to reduction factor R_{μ} using the intrinsic characteristics of the seismic demand and the SDOF system.

$$R_{\mu} = \frac{S_{ae}(T^*)}{S_{ay}} \qquad (1)$$
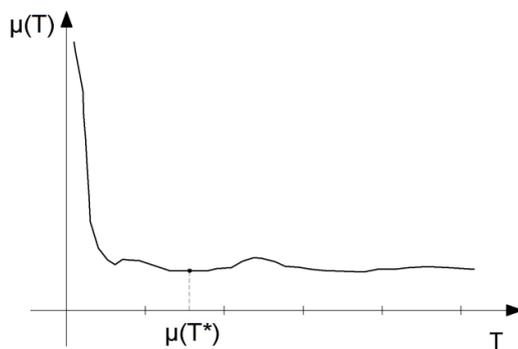
Figure 2a. The μ ductility spectra, for the constant value of R_{μ}
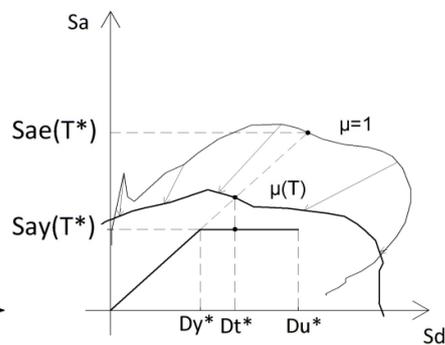
Figure 2b. Determination of the target displacement

The value of the target displacement D_t^* corresponding to the SDOF system is represented by the intersection of the constant period line which characterizes the elastic period T^* of the SDOF system.(fig.2b) .

The target displacement for the MDOF system can be obtained also analytically thorugh multiplying the displacement D^*_y by the first mode mass participation factor Γ and the ductility demand μ. (Eq. 2).

$$D_t = D^*_y \cdot \Gamma \cdot \mu \tag{2}$$

Although the graphical representation is not strictly necessary, it is recommended for visualization of quantities which govern the capacity and the demand, as represented on figure 2b.

3. Case study

The inelastic displacements of a reinforced concrete structure were determined by using the proposed method. The structure, the characteristics of which are presented in Fig. 3 a, b, c, has been previously used in paper [17]. As shown on Fig. 4a, the structure has two openings and six stories. The columns have height-variable section and reinforcement. A uniform loading of 20kN/m is applied to the beam elements. In order to avoid errors caused by distinct computational models, both static pushover analyses and dynamic time-history analyses were conducted using SeismoStruct [18] software package. A distributed inelasticity model is used to account for material inelasticity [18]. On the other hand, in terms of constant ductility inelastic spectra, seismic demands are generated by using Bispec software package [19]. The latter uses several inelastic response history analyses of SDOF systems for the assumed hysteretic behaviour. In the present study three different kinds of hysteretic models were used: a bilinear-plastic, a bilinear elastic, and a stiffness degrading model (Clough type) (Fig. 4 a, b, c). For the post-strain hardening and the viscous damping, a value of 5% was assumed.

Nivelul	Armare stâlpi
1	6Φ16
2	6Φ16
3	4Φ16
4	4Φ16
5	4Φ16
6	4Φ16

$f_y = 414 \cdot MPa$

$E_s = 210000 \cdot MPa$

$\dfrac{E_s}{E_i} = 0.009$

$f_{co} = 33 \cdot MPa$
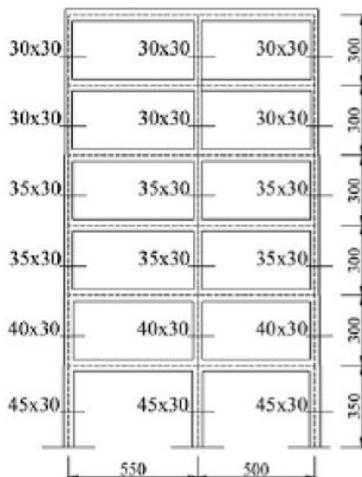
$\varepsilon_{co} = 0.0022$

$\varepsilon_{cu} = 0.0035$
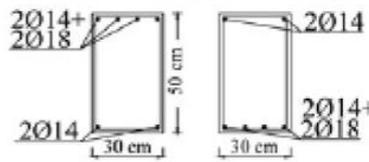
a.	b.	c.
Figure 3a. The geometrical configuration of the analyses structure [17]	Figure 3b. Beam and column reinforcement [17]	Figure 3c. Material characteristics [17]

The results obtained through the proposed procedure were compared with:
- The simplified design code-based displacement analysis procedure according to Romanian Seismic Design Code P100 [20](Annex E) and Ec8 [4] (chapter 4.3.4)
- The design code-based Nonlinear Static Pushover Analysis (NSPA) procedure (P100-Annex D, Ec8- Annex B)
- The „exact" results of Nonlinear Time-History Analysis (NTHA)
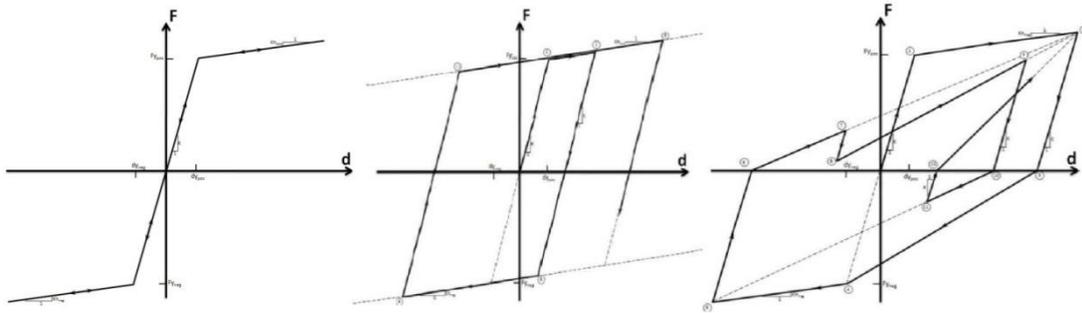
Fig. 4a. Bilinear elastic hysteretic model [19]

Fig.4b. Bilinear plastic hysteretic model [19]

Fig. 4c. Stiffness degrading hysteretic model (Clough) [19]

In order to facilitate the comparison with the code based pushover analysis method, the selected seismic records (Vrancea 1977, N-S component, INCERC recording station and Friuli 1976, N-S component, Tolmezzo-Diga Ambiesta station) were matched to the smoothed elastic design spectra characterized by $a_g=0.24g$ and $T_c=1.6s$ (P100), respectively $a_g=0.24g$ and $T_c=0.4s$, rock soil, type 1 (Ec8) (fig. 5 a, b). The matching was carried out using software package SeismoMatch [21]. The seismic records were also scaled to induce different levels of inelasticity, and consequently different ductility demands.

4. Results

The determination of target displacement through the graphical method is presented in Fig. 6. The numerical results are listed in Table 1. The results highlight that the simplified procedure highly overestimates the displacement demand. Another major inconvenient of this method is that it doesn`t offer information about the inelastic state of the structure. Observing the results, we can state furthermore, that from amongst the three methods of inelastic displacement determination, results yielded by static nonlinear analysis with directly generated inelastic spectra approximate best the results of the NTHA. Unlike the case of Friuli seismic record, the analyses conducted by the Vrancea demand yield different results for the three considered hysteretic rules (Fig. 6a1 6a2 and 6a3). The bilinear elastic hysteretic rule indicates too conservative results in the case of the Vrancea earthquake record, thus it`s use is not recommended. The bilinear plastic and stiffness degrading models produce almost exactly the same results as the NTHA. One can observe several cases where code based NSPA produces too conservative results (table 1). We consider that this discrepancy is due to the "static" character of the code-based method.

The latter observation emphasizes the importance of the "correct" tracing of the pushover curve.
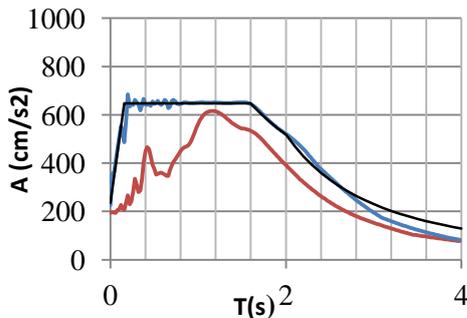
Figure 5a. Vrancea 1977 record (INCERC station), N-S component, matched to P100-2006 elastic design spectra, characterized by $a_g=0.24g$ and $T_c=1.6s$
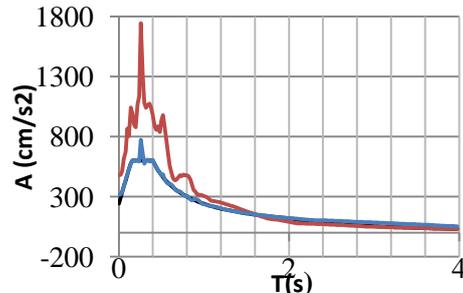
Figure 5b. Friuli 1976 record (Tolmezzo-Diga Ambiesta station), N-S component, matched to EC8 elastic design spectra, type A , soil rock, characterized by $a_g=0.24g$ and $T_c=0.4s$
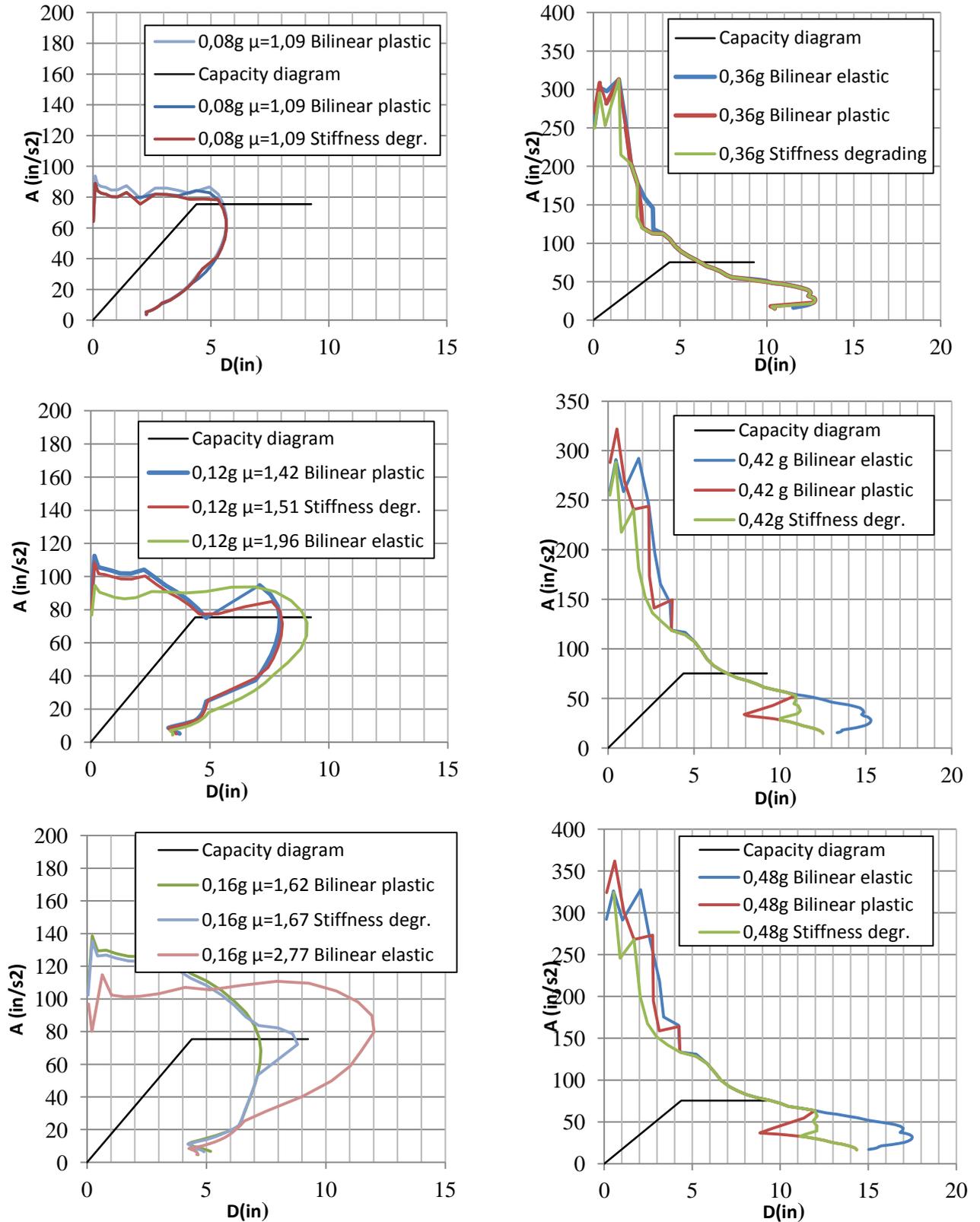
Figure 6. Determination of target displacement through the graphical method: Vrancea 1977 record matched to P100 elastic design spectra (left, top-to-bottom 6a1, 6a2, 6a3)/ Friuli 1976 record matched to Ec8 elastic design spectra (right, top-to-bottom, 6b1, 6b2, 6b3)

Table 1. Summary of the displacement results

	a_g	Graphical representa-tion	Simplified procedure (cm)	Code-based NSPA (cm)	NSPA with directly generated constant ductility spectra (cm)			Dynamic time- history Analysis (cm)
					Hysteretic model			
					Biliniar plastic	Biliniar elastic	Stiffness degrading	
Vrancea 1977 & P100	0,08g	Fig. 6a1	19,21	16,82	16.51	16.67	16.51	19,80
	0,12g	Fig. 6a2	28,81	25,60	21.51	29.69	22.88	21,20
	0,16g	Fig. 6a3	38,41	33,79	24.54	41.97	25.30	26,08
Friuli 1976 & Ec8	0,36g	Fig. 6b1	24,03	19,38	19,09			22,10
	0,42g	Fig. 6b2	28,30	22,57	25,60			25,50
	0,48g	Fig. 6b3	32,04	25,80	26,66			30,80

5. Conclusions

We presented thus an NSPA method, which uses directly generated inelastic spectra from specific earthquake recordings. The ductility demand is determined for the assumed hysteretic model through the inelastic response history analysis of the SDOF system. The apparent contradiction between the static feature of the procedure and the use of the nonlinear dynamic analysis of the equivalent SDOF system is justified by the extremely low computational demand and by the fact that a series of approximations necessary for the determination of the inelastic spectra are eliminated. [15] In the case of a seismic demand represented by an earthquake, the proposed method can be an alternative to the dynamic time-history analysis. If the seismic records are compatible with the elastic design spectra, the method can substitute the code based NSPA. The method also retains the advantage that capacity and demand can be graphically represented. The results have shown good method applicability in the case of the mid-range period structures (velocity sensitive region). An important advantage of the method regarding the code based NSPA is that it can take into account the characteristics of the specific earthquake record for the determination of the inelastic displacement demand. This attribute of the method can be useful especially in the acceleration and displacement sensitive spectral regions, where code based procedures tend to over- or underestimate displacement demands.

6. References

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