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Finite Element Method for the Stress Concentration Factor in Plates with Holes

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Abstract

In this article, there is an analysis of the influence of holes on the bearing capacity of reinforced concrete flat plate bent, showing that this influence depends on the position, size and shape of the hole, and depending on how plates are supported, reinforcement and loads. The first part is a presentation of how the dimensioning of reinforced concrete slab, with holes, after normative Eurocode 2, the provisions of the individualized plates reinforced one-way and plates reinforced in two directions. The described geometry of the support mode and loading of plates are studied. We have found, first of all, supported on two sides of the flat plate loaded with a uniformly distributed force. The goal, of rectangular shape was placed in different positions, at a variable distance from the edge. There have been maps and diagrams of efforts on the whole slab from the edge of the opening the axis of the opening. Analysis is presented in tabular form, from which we can find the maximum values, and reports between the the maximum and the field moments. The results are tabulated and mapped graphics together with relevant comments that allow us to formulate some conclusions about design principles and simple rules. Conclusions present some design rules for the dimensioning side strips of holes in plates supported on two sides and then expanded the rules for sizing the plate shell belts supported on four sides.

Rezumat

În acest articol, se face o analiză a influenţei golurilor asupra capacităţii portante a plăcilor plane încovoiate din beton armat, arătând că această influenţă este in funcţie de poziţia, mărimea şi forma golului, cât şi funcţie de modul de rezemare, armare şi încărcare al plăcilor. În prima parte se face o expunere a modului de dimensionare a plăcilor de beton armat, cu goluri, după prevederile normativului Eurocod 2, prevederile fiind individualizate pentru plăci armate pe o singură direcţie şi plăci armate pe două direcţii. În continuare se descrie geometria, modul de rezemare şi de încărcare a plăcilor studiate. S-au considerat, mai întâi, plăci plane rezemate pe două laturi, încărcate cu o forţă uniform distribuită. Golul, de formă dreptunghiulară s-a amplasat în diverse poziţii, la o distanţa variabila de marginea rezemată. S-au trasat diagramele de eforturi pe toată deschiderea, până la marginea golului, respectiv până în axul golului. Rezultatele analizei se prezintă sub forma tabelară, din care se pot afla valorile maxime cat şi rapoartele intre momenul maxim si cel din camp. Rezultatele tabelare şi graficele trasate sunt însoţite de observaţii pertinente care permit autorului tezei să formuleze unele concluzii referitoare la principii şi reguli simple de proiectare. Concluziile prezinta cateva reguli de proiectare pentru dimensionarea fâşiilor de bordaj ale golurilor la plăci rezemate pe două laturi şi apoi a extins aceste reguli pentru dimensionarea fâşiilor de bordaj la plăci rezemate pe patru laturi.

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1. Dimensioning of the plates with holes according to Euro code 2

The goals may influence the capacity of the plates depends on the position, size, hole shape, the way the supporting reinforcement, in one direction or two, and for the application of the loads.

1.1 <u>One-way reinforced slabs</u>

For plates reinforced on a single direction we have two cases:

a. when the size of the hole is less than 20% the span of the plate, corresponding the reinforcement hole is placed in the vicinity/edge of the opening, which extends the length of the anchorage;

b. if the size of hole is greater than 20% of the span of the plate (l_x) and the hole is placed at the midspan of the plates should be sized to an increased bending moment given by the following equation:

$$m_x = \left[0{,}125 + 0{,}19\frac{a}{l_x}\left(\frac{2b}{l_x}\right)^2\right] q l_x^2 \qquad (1.1)$$

and the reinforcement strips are disposed adjacent to the opening, the width of

$$b_m = 0{,}8 l_x - b \qquad (1.2)$$

On the direction of the distribution reinforcements on both sides of the hole we have two cases:

- if b/a < 0.5 then the dimensioning of the reinforcements on both sides of the hole when m_y is calculated and the plate support is articulated on three sides and free on one side.
- if b/1 > 0.4 then the reinforcements near the edges are dimensioned for the bending moment.

$$m_y = 0{,}125\, q\, a\, (a + 2b_m) \qquad (1.3)$$

where b_m is given by de 1.2.

The reinforcements near the edges of the hole are usually done by bending the bars or nets (mesh) or by using special rebar, these reinforcements are extended in the slab by:

$$c_i \geq \frac{a_i}{2} + l_{bd} \qquad (1..a)$$

where l_{bd} is the length.

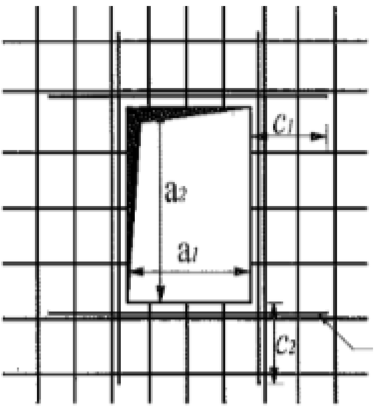


Figure 1-1

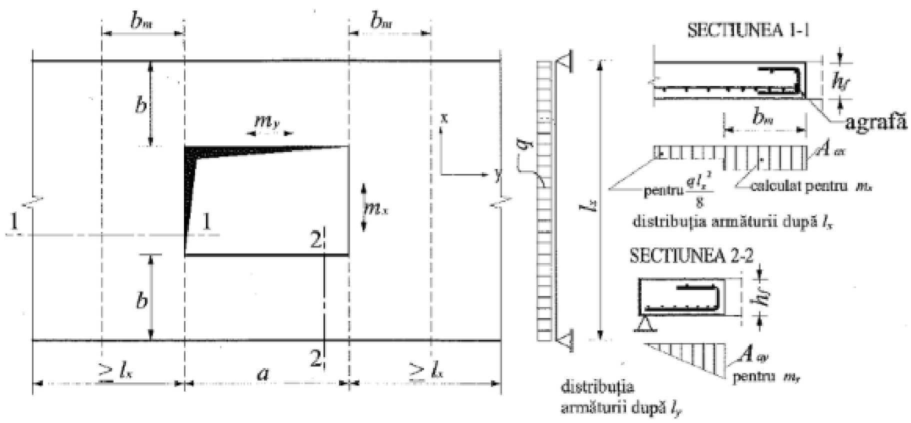


Figure 1-2

1.2 <u>Two-way reinforced slabs</u>

Reinforcements of slabs on two direction with holes of less than 20% of both sizes of the plate can be treated similar to the reinforcements in one way reinforced slabs. The rebar used in the vicinity of the hole should be extended by the length of anchorage.

A simplified solution for solving slabs reinforced on two ways for holes bigger than 20% of any size of the slab is treating it like four plates with articulate support for three of the sides. Rationally chosen boundary conditions may lead to most unfavourable reinforcements dimensioning. Additional rebar used in the vicinity of the hole are placed on the lower and upper parts from support to support. Both of the rebar placed lower and upper near the hole form a beam having the thickness equal to the one of plate, and when in the vicinity of the hole we have high loads, the thickness can exceed the one of the plate.

Possible incompatibilities that cannot be avoided using approximate methods of calculation of the loads, are taken over by the upper rebar. Lower rebar on the direction of the maximum stress continue to the supports without reducing the cross section.

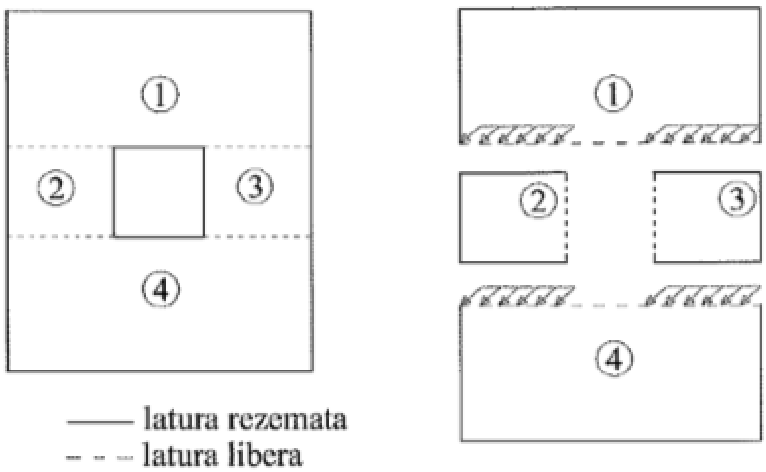


Figure 1-3

In the case of two-way reinforcements in an approximate representation of the slab, is considered broken down as shown in Fig. 1-3. Plates 2 and 3 are calculated like plates with support on three sides and free on the fourth, with the q load of plate. Plates 1 and 4 are calculated like plates with support on 3 sides as for the fourth side, we will load this sides with the reactions resulting from the calculation made for plates 2 and 3. Technology and methods of calculation allow solving the state plane stress problem through numerical methods. For this purpose finite difference method can be used (with some difficulty in imposing boundary conditions) but more efficient is the finite element method (FEM).

2. The studied geometry and loading elements

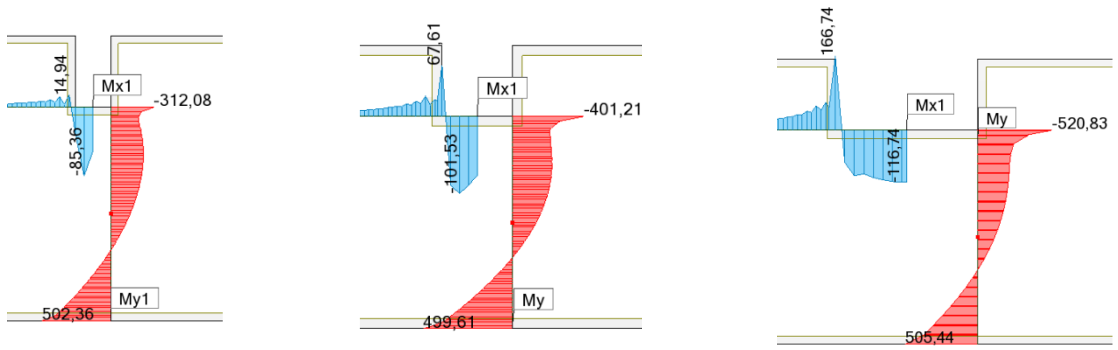
From those showed in the previous paragraph, it appears that the loading problem of the plates with holes are used for the particular case of the hole opening disposed at the midspan. In general cases the plates supported on four sides, the approximate solutions using the following decomposition plate solutions in very rare cases in the literature. In order to a more rigorous analysis and consideration of various geometric parameters that characterize the plates with holes, such as, in particular, the ratio between the size of the gap, the ratio between the size of the plate opening and opening position and the opening position in relation to the opening, it was proposed to establish the FEM stress to be used in the local size of the local dimensions of these plates.

The structural model are plane plates supported on two sides (Fig 6.4), with the opening of 6 and the thickness gp=0,2 m (gp/l=1/30), under load with 100 kN/m2. To expand the influence of free edges from x=±∞ (shown, Saint Venant's principle up to a distance approximately equal to 1) length L of the plate was considered of 25 m (>4l, and the distance from the edge of the hole to the free edge >4 1,5l). The support conditions have been studied hinge-hinge, fixed-fixed, and fixed-hinge. It was considered an hole of rectangular form axb, (with the side parallel with the edge support) located on the side of y=0 to the mid-span of the symmetric support (hinge-hinge, fixed-fixed) and to the edge of y=l in the fixed-hinge case. Stress diagrams in the vicinity of the holes.

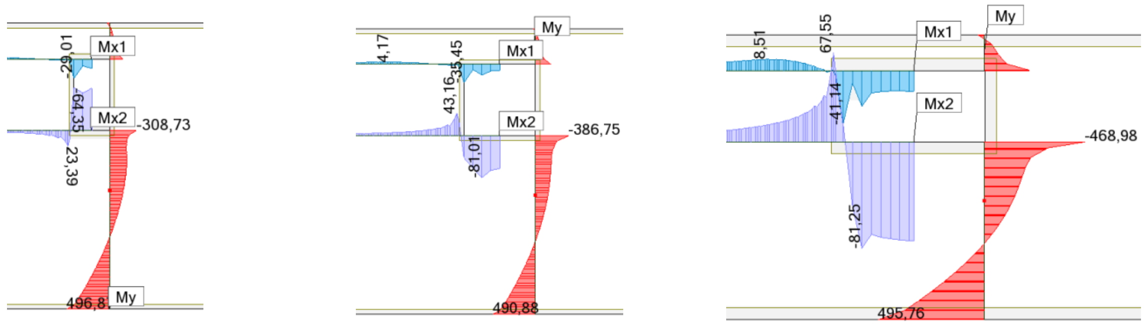
The following figures are shown some typical diagrams obtained with FEM for the three support conditions and reinforcement:

- for the plate with the support fixed-hinge Figure 2-1 A-I/1 and 2

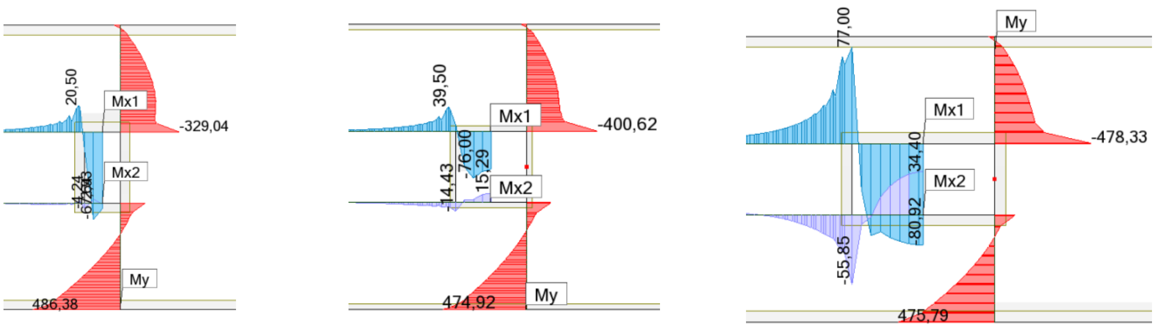
a/b=1/2	a/b=1	a/b=2



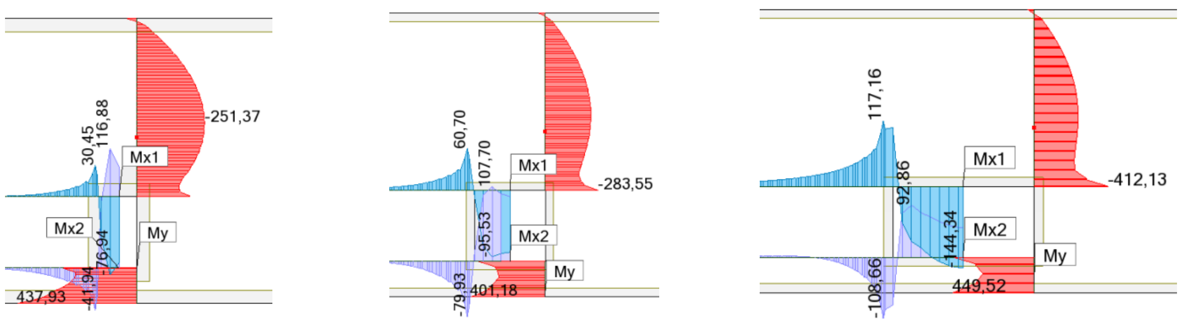
Hole with b=l/4, at d=l/8, Mx and My bending moment diagrams (kNm/m)



Hole with b=l/4, at d=l/4, Mx and My bending moment diagrams (kNm/m)



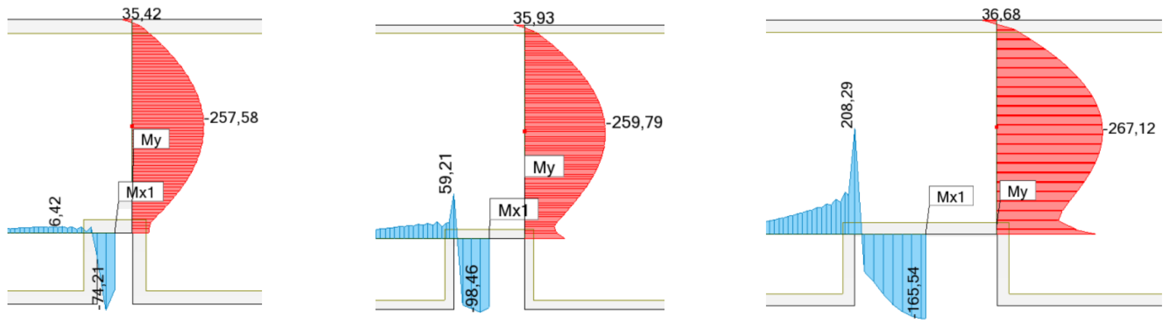
Hole with b=l/4, at d=l/2, Mx and My bending moment diagrams (kNm/m)



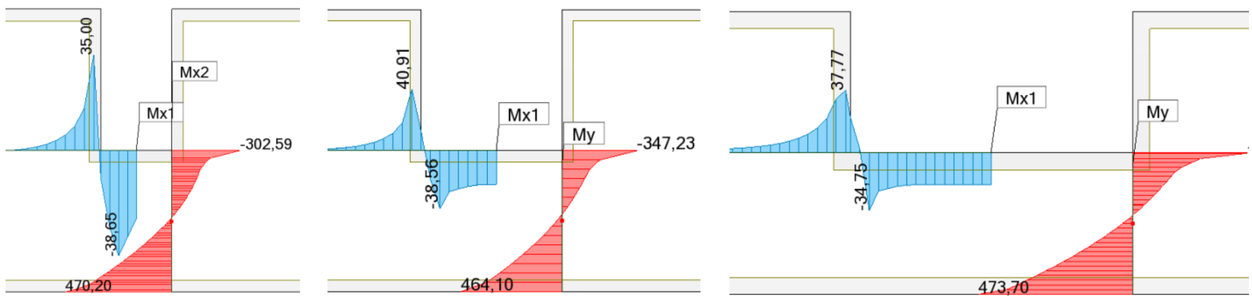
Hole with b=l/4, at d=3l/4, Mx and My bending moment diagrams (kNm/m)

Figure 2-1 A-I/1 – Mx and My bending moment diagrams for hinge-fixed support

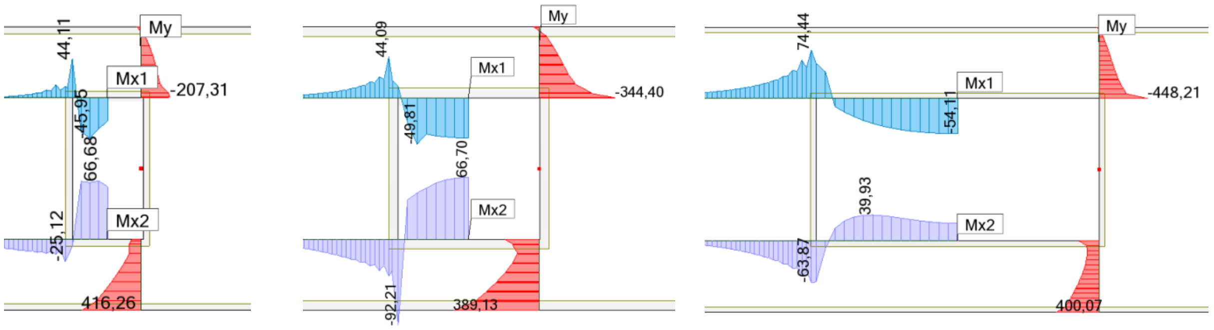
a/b=1/2	a/b=1	a/b=2



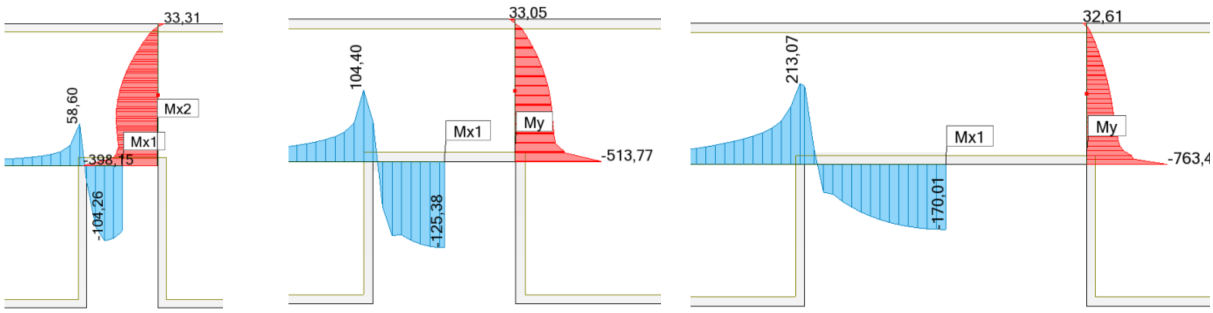
Hole with b=l/4, at d=7l/8, Mx and My bending moment diagrams (kNm/m)



Hole with b=l/2, at d=l/4, Mx and My bending moment diagrams (kNm/m)



Hole with b=l/2, at d=l/2, Mx and My bending moment diagrams (kNm/m)



Hole with b=l/2, at d=3l/4, Mx and My bending moment diagrams (kNm/m)

Figure 2-1 A-I/2 – Mx and My bending moment diagrams for hinge-fixed support

Let us presume that near the hole, on a width equal to λ (Figure 2-2) additional reinforcement demanded by the presence of the hole is made.

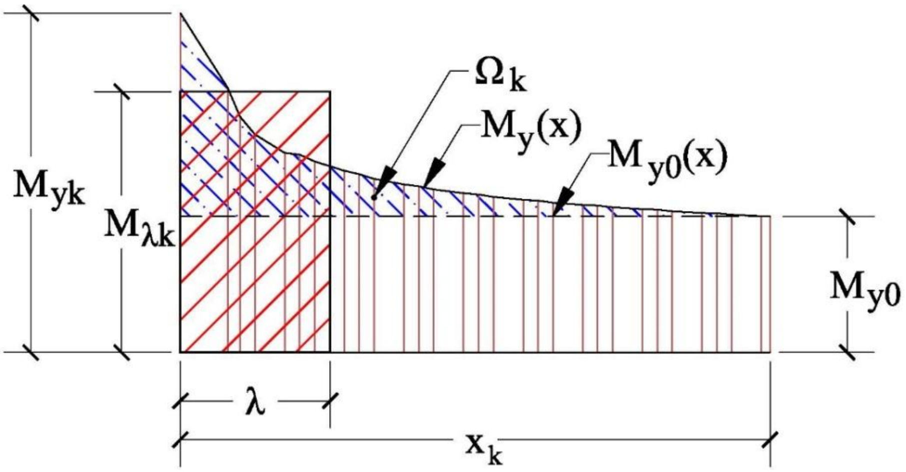


Figure 2-2

Consider Ω_k the area on the influence length x_k between $M_y(x)$ and $M_{y0}(x)= M_{y0}$=constant. This equals

$$\Omega_k = \int_{x_k} M_y(x)\,dx - x_k \cdot M_{y0} \tag{2.1}$$

Admitting, approximate, that after the stress redistribution, on a band with λ width near the hole (called side strip), the bending moment $M_{\lambda k}$ is equal to M_{lim} (moments on length units). To equivalent the moment increment Ω_k form the area of influence it is necessary that the area $\lambda M_{\lambda k}$-λM_{y0} be equal to Ω_k:

$$\lambda\left(M_{\lambda k} - M_{y0}\right) = \Omega_k \tag{2.1.a}$$

or

$$\lambda M_{\lambda k} = \Omega_k + \lambda M_{y0} \tag{2.2}$$

The reinforcement $A_{a\lambda}$ from the border strip with λ width must be designed so that $M_{cap}=M_{lim}=\lambda M_{\lambda k}$ (Figure 2-3).

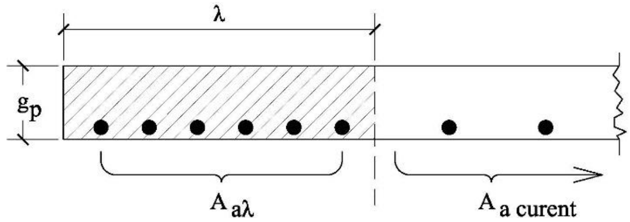


Figure 2-3 Border strip reinforcement

The reinforcement factor of the side strip should satisfy the convention $\rho<\rho_{max}$ required by the design regulations.

3.Results analysis

In table 3-1 are shown the main results that intervene in the analysis, namely:

1.- The values M_{yk} and M_{y0}

2.- The ratios >M_{yk}/M_{y0}

3.- Area of influence z_k

4.- The value $\Omega_k = \int_{x_k}\left[M_y(x) - M_{y0}\right]dx$

5.- The values M_λ – representing the bending moment that the side strip must endure if its width is $\lambda = 1{,}5gp = 0{,}30m$, that is $M_\lambda = \Omega_k/\lambda + M_{y0}$

6.- The ratios M_λ/M_{y0} for the three support cases analyzed.

All the values are set for an opening length l=6,0 m and a (conventional) load by 100 kN/m^2.

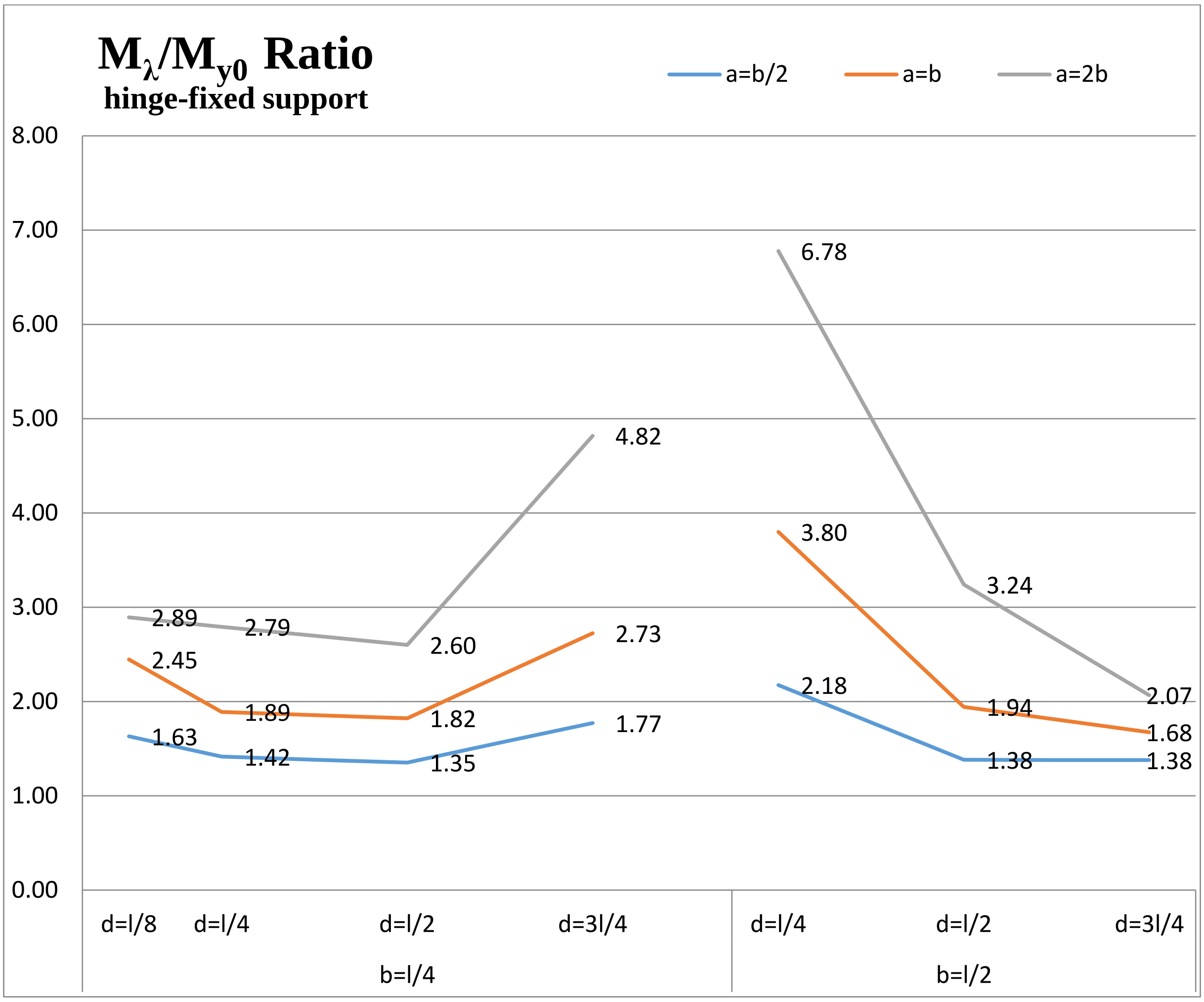
Monitoring the enlargement of the area influenced by the presence of the hole, we observe that is limited at x_k<4,30m, which represents 0,72 from the opening length l. Load location confirm Barré de Saint Venant's principle.

M_{yk}/M_0 ratios variations (for conventional λ equal to 0,30m) are presented below.

Table 3-1– Hinge-fixed support, values M_{yk},M_{y0}, M_{yk}/M_{y0}, z_k, Ω_k, M_λ and M_λ/M_{y0}

Value	b/l	$\frac{1}{4}$					$\frac{1}{2}$		
	d/l a/b	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
M_{yk} (kNm/m)	1/2	-312,08	-308,73	-329,04	-194,76 264,84	-58,86	-302,59	-303,80	-398,15
	1,0	-401,21	-386,75	-400,62	-286,55 358,40	-128,40	-347,33	-354,4	-543,77
	2,0	-520,83	-468,98	-478,33	-412,33 441,47	-265,29	-355,07	-448,21	-763,49
M_{y0}	all	-224,83	-254,12	-254,12	-141,96 194,3	-0,1	-224,83	-224,81	-227,17
M_{yk}/M_{y0}	1/2	1,39	1,21	1,29	1,37	588,6	1,33	1,35	1,75
	1,0	1,78	1,52	1,58	2,02	1284	1,53	1,58	2,39
	2,0	2,32	1,85	1,88	2,91	2652,9	1,56	1,99	3,36
z_k (m)	1/2	2,07	1,50	1,02	1,94	2,42	1,19	1,27	2,31
	1,0	2,80	2,07	2,70	2,24	3,29	2,29	2,11	3,13
	2,0	2,98	3,06	2,42	2,57	2,75	2,64	3,12	3,86
Ω_k (kNm)	1/2	42,56	31,69	27,05	32,87	33,93	80,17	25,79	25,83
	1,0	97,55	67,93	62,87	73,49	68,04	191,02	63,71	46,03
	2,0	127,82	136,77	122,05	162,68	137,79	394,02	151,24	72,81
M_λ (kNm/m)	1/2	366,68	359,75	344,28	251,55	113,48	494,33	310,52	313,42
	1,0	549,98	480,56	463,67	386,96	227,39	864,11	437,27	380,72
	2,0	650,92	710,02	660,96	684,17	458,64	1540,7	729,12	469,60
M_λ/M_{y0}	1/2	1,63	1,42	1,35	1,77	1134,8	2,18	1,38	1,38
	1,0	2,45	1,89	1,82	2,73	2273,9	3,80	1,94	1,68
	2,0	2,89	2,79	2,60	4,82	4586,4	6,78	3,24	2,07

Below, the M_{yk}/M_{y0} and M_λ/M_{y0} ratios variations charts are shown, and by analyzing them, we drawn the observations presented at each chart.



From the chart above result that simultaneous with the increase of b/l ratio, M_λ/M_{y0} bending moments ratios are bigger. Overall, average M_λ/M_{y0} ratio when b=l/4 is 2,35, and when b=l/2= is 2,72, we notice a average difference by 15%.

In the variations of a in relation to b, it's clearly noticed that the smaller the ratio between a and b is, the smaller M_λ/M_{y0} ratio is also, and comparing average percentage we could say that for a=b, M_λ/M_{y0} ratio is 31% bigger than a=b/2 and with 35% smaller than a=2b. We could also say that in the case of variation of a in a relation to b we obtain charts of similar shapes.

For the same reasons as in the precedent M_{yk}/M_{y0} ratio, we chose to omit M_λ/M_{y0} values, values that are very big compared to the rest of the values due to M_{y0} values that converge to 0.

In the chart above we noticed the influence of the hole eccentricity in regard to the edge d, which, in the case of bigger holes (b=l/2) is quite obvious, i.e. the value of M_λ/M_{y0} ratio decrease as the hole stand away from the de hinged edge and approach of the fixed edge with minimum values of the ratio when the hole is on the fixed side. In the case of smaller holes (b=l/4) we noticed that the M_λ/M_{y0} ratio decrease at the first displacements towards the center, and then begin to increase from the middle as approached to the fixed edge.

6. References

1. **Heyman, Jacques.** *The Science of Structural Engineering.* Londra : Imperial College Press, 1999. ISBN 1-86094-189-3.

2. **Ille, Vasile și Bia, Cornel.** *Rezistența Materialelor vol. I.* Cluj-Napoca : Institutul Politehnic Cluj, 1969.

3. **Bia, Cornel, Ille, Vasile și Soare, M.V.** *Rezistența Materialelor și Teoria Elasticității.* București : Editura Didactică și Pedagogică, 1983.
4. **Girkmann, Karl.** *Flächentragwerke.* Wien : Springer Verlag, 1956.
5. **Boresi, Arthur P. și Schmidt, Richard J.** *Advanced Mechanics of Materials.* s.l. : John Wiley & Sons. INC, 2003.
6. **N. I., Bezuhov.** *Teoria Elasticității și Plasticității.* București : Editura Tehnică, 1957.
7. **Gere, J. M.** *Mechanics of Materials.* s.l. : Thomson Learning, 2004.
8. **Hutte.** *Manualul inginerului.* Bucuresti : s.n., 1949.
9. **Pilkey, Walter D.** *Peterson's Stress Concentration Factors.* New-York : John Wiley & Sons, 2nd Ed 1999. ISBN 0-471-53849-3.
10. **Bia, Cornel.** *Teoria Elasticității și Plasticității.* Cluj-Napoca : I.P.C.N., 1980.
11. **Soare, M.V.** *Rezistența materialelor. Curs și aplicații.* București : Institutul de Construcții București, vol.I 1978, vol.II 1978, vol.III 1982.
12. **Mazilu, P., Posea, N. și Iordăchescu, E.** *Probleme de rezistența materialelor* . București : Editura Tehnică, vol. I 1969, vol.II 1975.