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Evaluation of deflections for PFRP-RC hybrid beams with complete and partial shear connection

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Abstract

The current paper presents various analytical models suitable for the evaluation of hybrid beam deflections, ranging from simple to complex expressions, which can be applied in common design practice. The formulations are based primarily on the Euler-Bernoulli composite beam theory and on the Stüssi–Granholm–Newmark–Pleshkov interlayer slip model, completed by shear deflection contributions based on the Timoshenko beam theory. The analytical expressions rely on the degree of shear interaction and connection stiffness, and can be independent of the loading and supporting conditions. The theoretical models were validated successfully against experimental data found in literature. Conclusions are drawn concerning the recommended formulations for use in practice.

Keywords: FRP profile; hybrid beam; deflection; partial interaction; composite action; interlayer slip; analytical study; composite materials; flexural behavior.

1. Introduction

The past three decades have witnessed a continuous increase in the use of composite materials in infrastructure and building projects. Currently there is a wide array of composite shapes and corresponding construction technologies varying from laminates, reinforcement bars and textile fabrics to structural pultruded profiles.

Researchers have recently combined pultruded FRP profiles (PFRPs) with a more traditional material, reinforced concrete (RC), to obtain new hybrid types of elements with superior strength and stiffness characteristics [1]. As these hybrid structural elements span greater lengths their behavior starts to be governed not by their ultimate capacity but by service state limitations such as admissible deflections.

Major design codes such as Eurocode 4 or AISC 360-10 take into consideration the degree of shear interaction in the evaluation of flexural deflections for beams, but only from the perspective of the capacity of installed connectors. However, even in hybrid beams with a sufficient number of connectors, larger deflections than predicted were observed during experimental trials which were later imputed to the flexibility of the connection system [2].

Hence, this study proposes a simplified analytical method for evaluating the deflections of hybrid PFRP-RC beams under short-term loading, in both complete and partial interaction conditions. The method is based on the Euler-Bernoulli composite beam theory, Timoshenko beam theory and

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elastic partial interaction theory. It can be applied indifferent to the loading and supporting conditions.

2. Analytical models

The two following sections present analytical expressions for evaluating deflections under complete or partial shear interaction conditions for a hybrid beam model comprised of an I-shaped profile and a rectangular reinforced concrete slab. The formulations can be extended to other prismatic cross-section geometries which possess a vertical axis of symmetry. Calculation hypotheses are listed together with generally valid or specific case equations. The evaluation of deflections is performed under the elastic range of the beam's constitutive materials.

2.1 Complete shear interaction

The elastic analysis of the PFRP-RC hybrid beam is performed assuming the following hypotheses: the whole width of the concrete slab is effective; there is no longitudinal slippage between the concrete slab and the PFRP profile; there is no vertical separation between the concrete slab and the PFRP profile; and the Euler-Bernoulli hypothesis is valid.

Since the ratio between the longitudinal elastic modulus and the shear modulus of the composite is relatively low, a hybrid beam is susceptible to a significant increase in deflection due to shear deformations, a trait especially true for stockier members. Therefore, the elastic curve that describes the deflected shape of a hybrid element is based on the Timoshenko beam theory, and is a function of its flexural rigidity, denoted EI, and transverse shear rigidity, κAG. The solution for the total deflection $w_t(x)$ with appropriate boundary conditions leads to the general expression as a sum of the deflection due to bending deformation $w_b(x)$ and the deflection due to shear deformation $w_{sh}(x)$, at a position x along the beam:

$$w_t(x) = w_b(x) + w_{sh}(x) = \frac{f(x)}{EI} + \frac{g(x)}{\kappa AG} \tag{1}$$

where $f(x)$ and $g(x)$ are functions that depend on loading and boundary conditions. For determinate beams, simply supported over a span distance L, loaded with a concentrated force Q at their midspan, as shown in Fig. 3, the functions are equal to:

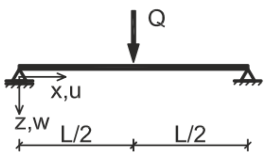

Figure 3. Case example of a 3-point bended simply supported hybrid beam.

$$f(x) = \frac{Qx}{48}(3L^2 - 4x^2)$$
$$g(x) = \frac{Qx}{2}$$

For composite members with complete shear interaction, the flexural rigidity EI can be computed from the following relation:

$$EI_{co} = EI_0 + EA^* d_c^2 \tag{2}$$

where $EI_0 = E_c I_c + E_p I_p$ is the flexural rigidity when no connection is present (no shear interaction), $EA^* = (E_c A_c \cdot E_p A_p)/(E_c A_c + E_p A_p)$, and d_c is the distance between the centroids of the cross-sections of the two materials, A_c and A_p.

After cracking of the concrete slab occurs, a reduced flexural rigidity of the slab should be obtained by taking into account only the concrete area working under compression.

As for composite steel-concrete beams, it is assumed that the concrete slab does not contribute greatly to shear capacity and therefore it is considered, in a conservative approach, that only the web of the profile carries the shear load. For homogeneous pultruded shapes, having the same properties in the flanges and webs of the profile, the transverse shear rigidity can be approximated as:

$$\kappa AG = \kappa A_p G_p \approx A_{web} G_{web} \tag{3}$$

The approximation presented above is best suited for analytical design because often times the calculation of the shear coefficient κ is cumbersome even for simple cross-sections of profiles.

2.1 Partial shear interaction

An approximate static analysis and design procedure based on the Stussi-Granholm-Newmark-Pleshkov model is presented for the partial composite action theory of Euler-Bernoulli beams with interlayer slip. The analytical procedure can be used for hybrid beams that have various load arrangements and supporting conditions, and it is based on the following hypotheses: discrete connections are replaced by a linearly elastic medium, for situations in which forces on connectors do not exceed about half their ultimate strength; shear at the interface is proportional to slip; the PFRP profile and concrete slab have the same curvature and rotation at the same section; material behavior of PFRP and concrete is linear elastic; reinforcement bars in the slab don't provide vertical shear resistance and concrete tension capacity is neglected.

Because of the numerous factors which influence the determination of the shear connector modulus, obtaining a simple but reliable design formulation presents difficulties. Where data from experimental tests is unavailable various studies propose empirical expressions to be used such as the one in [3] for steel-concrete composite beams with steel studs:

$$K_s = \frac{P_{max}}{d_s(0.16 - 0.0017 f_c)} \tag{5}$$

where P_{max} is the maximum capacity of a connector, d_s its diameter, and f_c represents the concrete slab's compressive strength.

The suggested method in this study for determining the bending deflection of hybrid beams with partial interaction is based on calculating a reduced, effective flexural rigidity EI_{eff} and hence an increased effective bending deflection w_b^{eff} using a proposed dimensionless parameter denoted ξ which reflects the influence of the connection's flexibility over the structural response of the element, by affecting the complete flexural rigidity EI_{co} and complete bending deflection $w_b^{co}(x)$.

$$EI_{eff} = \frac{EI_{co}}{1 + \xi} \tag{6}$$

$$w_b^{eff}(x) = w_b^{co}(x) \cdot (1 + \xi) \tag{7}$$

One of the advantages of using this type of formulations in design is, besides its simplicity, the fact that the results are not sensitive to load type and supporting conditions and thus the expressions can be regarded as generally valid. The effects of partial interaction over shear deflection contributions are not treated herein, considering only the case of shear rigid connections.

The differential equations [4] which govern the flexural behavior of a hybrid beam with slip occurring at the interface are:

$$\frac{\partial^6}{\partial x^6} w_b(x) - \alpha^2 \frac{\partial^4}{\partial x^4} w_b(x) = \frac{1}{EI_0} \frac{\partial^2}{\partial x^2} q(x) - \alpha^2 \frac{1}{EI_{co}} q(x) \tag{8}$$

$$\frac{\partial^2}{\partial x^2} N(x) - \alpha^2 N(x) = -\frac{K_s d_c}{s \cdot EI_0} M(x) \tag{9}$$

where s is the longitudinal spacing of the connectors, and α is obtained from:

$$\alpha = \sqrt{\frac{K_s d_c^2}{s} \frac{EI_{co}}{EI_0(EI_{co} - EI_0)}} \tag{10}$$

Exact solutions to differential Eqs. (8) and (9) can be obtained by considering appropriate boundary conditions for specific cases, where at certain coordinates along the beam the deflection, slope or curvature have a known value. For the case exemplified in Fig. 3, a simply supported 3-point bended hybrid beam, the exact solution determined is:

$$w_b(x) = w_b^{co}(x) + \frac{Q}{\alpha^3 EI_{co}} \left(\frac{EI_{co}}{EI_0} - 1\right) \left[\frac{\alpha x}{2} - \sinh\left(\frac{\alpha L}{2}\right) \frac{\sinh(\alpha x)}{\sinh(\alpha L)}\right] \tag{11}$$

Correspondingly, the expression for the exact dimensionless parameter ξ_{EX} was obtained from Eqs. (7) and (11), where $c = x/L$ represents the relative longitudinal coordinate:

$$\xi_{EX}(c) = \frac{24}{(\alpha L)^3 (3c - 4c^3)} \left(\frac{EI_{co}}{EI_0} - 1\right) \left[\alpha L c - \operatorname{sech}\left(\frac{\alpha L}{2}\right) \sinh(\alpha L c)\right] \tag{12}$$

Simplified expressions for calculating the effective bending deflection of various types of composite members with partial interaction have been suggested by different authors. By applying the proposed method from the current study, their expressions were converted to the dimensionless parameter ξ from Eqs. (6) and (7).

For steel-concrete composite beams, Nie and Cai [2] proposed after various simplifications the following generalized formulation which was argued to be close to the response of the uniformly loaded beam case:

$$\xi_{NC} = \frac{12}{(\alpha L)^2} \left(\frac{EI_{co}}{EI_0}\right) \left(0.4 - \frac{3}{(\alpha L)^2}\right) \tag{13}$$

For the same type of composite element, Wang [5] suggested in his study a different expression which actually represents the exact solution for the maximum deflection of a simply supported beam under uniform loading conditions, considering partial interaction. From his results it was possible to compute:

$$\xi_W = \frac{76.8}{(\alpha L)^4} \left(\frac{EI_{co}}{EI_0} - 1\right) \left[\frac{(\alpha L)^2}{8} + \operatorname{sech}\left(\frac{\alpha L}{2}\right) - 1\right] \tag{14}$$

Girhammar [6] proposed a different method in his work which focuses on composite timber-concrete beams with incomplete shear connection. His studies aimed to adjust the so called "Gamma method" offered in Annex B of Eurocode 5 – Design of timber structures, by adding two parameters μ and μ_{co} which take into consideration the effective beam length of the problem, under partial and complete shear transfer mechanisms. The same author investigated the differences between buckling coefficients μ and μ_{co} and concluded that with the exception of the pinned-clamped supporting case the two are practically identical and that for a simply supported composite beam $\mu = \mu_{co} = 1.00$. In fact, the expression provided in Eurocode 5 represents the exact solution of the flexural rigidity of a simply supported beam with a sinusoidal load distribution. Using Eqs. (6) and (7), the dimensionless parameter obtained is:

$$\xi_G = \frac{\mu^2}{\mu_{co}^2}\left(\frac{EI_{co}}{EI_0} - 1\right)\left[1 + \left(\frac{\mu}{\pi}\right)^2 (\alpha L)^2\right]^{-1} \tag{15}$$

3. Parametric study

In what follows a comparative analysis has been made in order to emphasize the differences between the exact formulation for effective bending deflection and the previous simplified expressions provided by varying key parameters of the equations. The dimensionless parameter ξ which characterizes the influence of the partial composite action in a hybrid beam's equation of deflection is notably dependent on two other dimensionless factors: the composite action parameter αL and the relative bending stiffness parameter represented by the ratio EI_{co}/EI_0. Since the ratio is independent of the connections flexibility a constant value was set to 2.5 for the rest of the comparative analysis. The value represents an average of ratios obtained from experimental data found in literature for GFRP-RC hybrid beams with mechanical connections. Figure 5 plots the variation of the ratio between the effective and maximum complete bending deflection $\zeta = w_b^{eff}(x)/w_b^{co}(L/2)$ in function of relative longitudinal coordinates x/L, for the symmetric static case illustrated in Fig. 3. Values are plotted considering full composite action and low partial interaction with $\alpha L = 5$ (exact and simplified formulations).

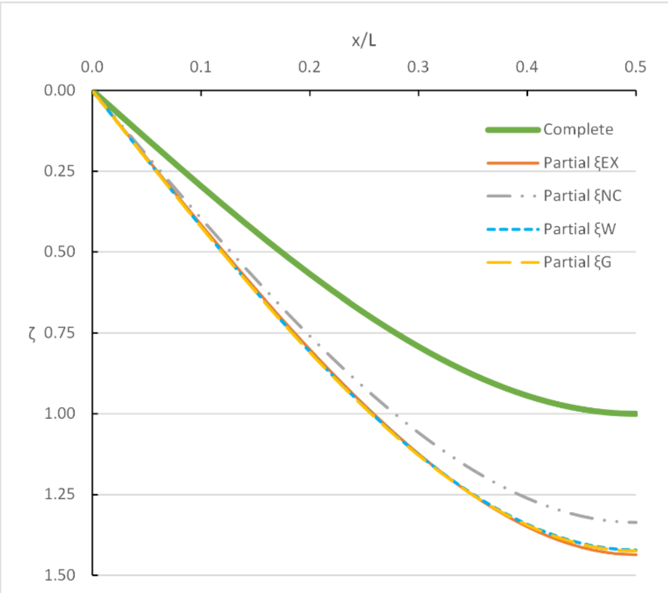

Figure 5. Variation of complete and partial relative deflections to relative coordinates.

As expected, a flexible connection between the two composing materials produces greater deflections than considering complete interaction, with ζ larger than unity. Simplified Eqs. (14) and (15) lead to closer values of the exact term ξ_{EX} than Eq. (13) for ξ_{NC}.

To better distinguish the advantages of one method over the other or the eventual errors that a simplified calculation can produce, Fig. 6 charts, for three partial interaction situations, the variation of ξ in function of relative longitudinal coordinates x/L. It is noticed that as the connection's stiffness increases, the greater the αL, the three simplified formulation merge with the response of the exact solution. For low interaction cases, in this specific scenario, the variation of ξ_{EX} is smaller than 5% with respect to the constant values of ξ_W and ξ_G.

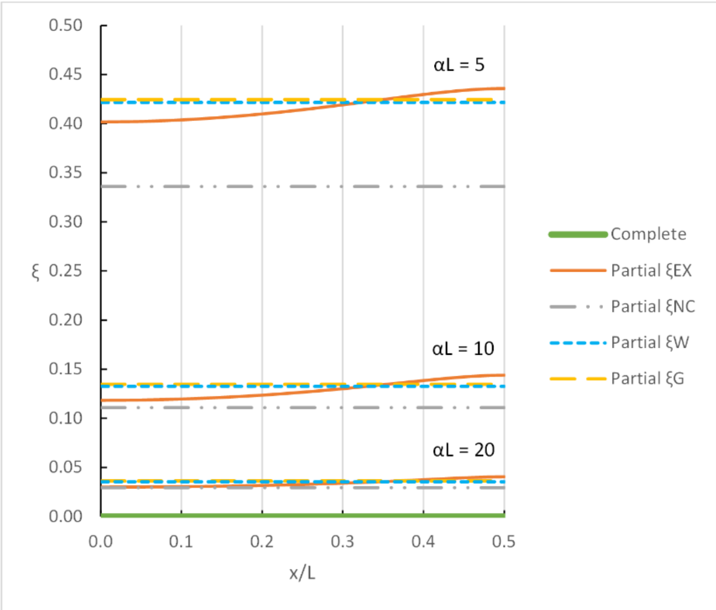

Figure 6. Variation of partial interaction parameter ξ to relative coordinates.

4. Validation of models

In the last part of the investigation the analytical models were validated against experimental data found in literature [1,7]. Ultimate limit capacities were evaluated using the relations suggested in [8].

Using the relations provided in section 2 the main parameters of partial interaction presented in Table 2 were computed for each specimen. Values for ξ were obtained from the exact solution and simplified Eq. (15) which best fitted the response for the calculation of maximum deflection of each specimen.

Table 2**:** Parameters of partial interaction for hybrid beam specimens

Specimen	EI_{co}/EI_0	αL	EI_{eff}/EI_{co}	ξ_{EX}	ξ_G	$\Delta\xi$ (%)
HB1	2.50	9.65	0.87	0.153	0.144	-5.9%
HB3	2.54	3.73	0.61	0.636	0.639	+0.5%
B7	2.92	7.73	0.79	0.282	0.272	-3.5%

Data for αL suggests that the hybrid beams had a low to medium degree of composite action. The stiffness ratios indicate a reduction of complete flexural stiffness of 13 to 39%. The difference $\Delta\xi$ between the exact and simplified method of determining ξ is negligible and therefore the approximate result was used in the remaining operations.

So as to isolate the dependency of a hybrid beam's maximum deflection to the connection's flexibility, in what follows the shear stiffness was calculated from an equivalent elastic finite element model of the specimen with complete shear interaction.

The maximum deflections of the hybrid beams were obtained at the Ultimate Limit State (ULS), using Eqs. (1), (2) and (7) and then compared to the experimental data reported in the studies. Table 3 summarizes the experimental values F_{EXP} and δ_{EXP}, the analytical values considering complete interaction δ_{CO}, partial interaction values δ_{EX}, δ_G, δ_{WEB} with ξ_{EX}, ξ_G where κAG is determined from either FEM analyses or Eq. (4); and the corresponding percentile differences versus the experimental data.

Table 3**:** Maximum total deflections considering various hypotheses at ULS

Specimen	**F_{EXP}** (kN)	**δ_{EXP}** (mm)	**δ_{CO}** (mm)	**δ_{EX}** (mm)	**δ_G** (mm)	**δ_{WEB}** (mm)	**Δ_{CO}** (%)	**Δ_{EX}** (%)	**Δ_G** (%)	**Δ_{WEB}** (%)
HB1	181.0	92.81	78.91	86.76	86.27	86.27	-15.0%	-6.5%	-7.0%	-7.0%
HB3	296.0	20.98	11.21	18.47	18.49	25.62	-46.6%	-12.0%	-11.9%	+22.1%
B7	70.9	70.38	52.85	61.29	60.84	63.57	-24.9%	-12.9%	-13.6%	-9.7%

As seen from Table 3 and Fig. 9 and 10, the proposed method predicts fairly well the maximum deflections for the hybrid beams under investigation, both at the Ultimate Limit State and at 50% of it. Results considering partial interaction provide higher accuracy in comparison with the evaluated deflections under complete interaction. At ULS slightly lower values were obtained due to the fact that concrete has a profound nonlinear response closer to its maximum strength. The analysis also reveals that considering only the shear stiffness of the profile's web, obtained deflections are on the safety side of the design, but a bit further from the experimental data.

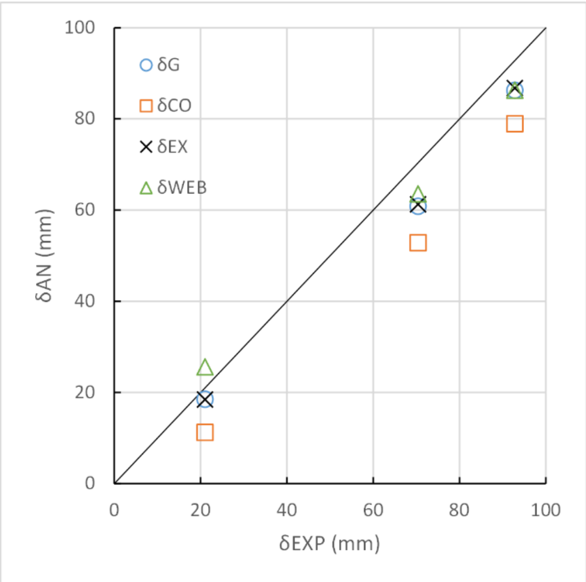

Figure 9. Analytic deflection δ_{AN} versus experimental deflection δ_{EXP}, at maximum capacity (ULS).

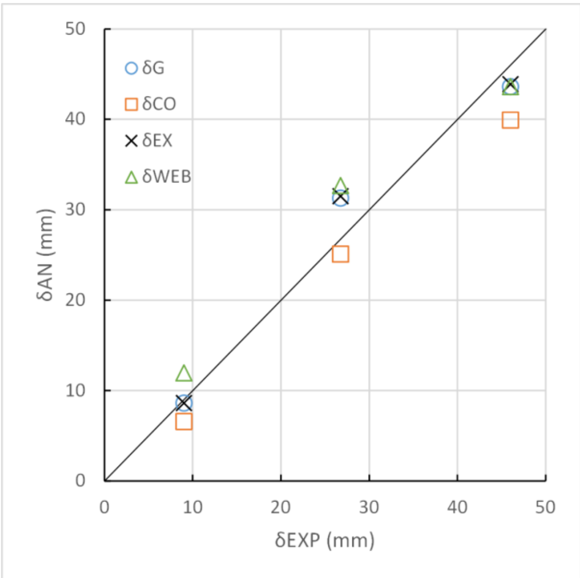

Figure 10. Analytic deflection δ_{AN} versus experimental deflection δ_{EXP}, at 50% of the maximum capacity.

5. Conclusions

The current study examined various analytical models suitable for the evaluation of PFRP-RC hybrid beam deflections which can be applied in common design practice. Partial interaction effects were also taken into account in function of the connection's flexibility, expressed through a dimensionless parameter. The advantage of the proposed method is the fact that it can be applied independent of the loading and supporting conditions and in an easy and direct manner. After a parametric study the closest approximate solution to the exact method was chosen and validated successfully against experimental data found in literature. The study demonstrated also that even in hybrid beams with a complete degree of shear interaction, slip effects should not be neglected as they can lead to substantially higher deflections and reduced flexural stiffness.

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