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Numerical and Analytical Analysis of Foundation Behavior on Soil Reinforced With Rigid Inclusions

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Abstract

Soil reinforcement by means of rigid inclusions is an efficient and economical solution especially for sites with medium soils where shallow foundations would lead to unacceptable settlements whereas deep foundations would be uneconomical. Although this type of reinforcement solution has been the subject of many studies, there are few references to the effect that the rigid inclusions have on the structural behavior of the foundation. This article analyzed a hypothetical situation in which rigid inclusions were used in order to reduce settlements. The focus was aimed at the stresses induced in the foundation by the presence of the rigid inclusions and the possibility of considering this influence in the foundation design. The problem was firstly studied through a unit cell model in the finite element analysis program, Plaxis 2D. The resulting foundation bending moments were then verified through an analytical approach. Finally, the problem was analyzed with a Winkler type model with differentiated subgrade reaction coefficients, in the GEO5 – Plate program. After a calibration of the elastic parameters, the obtained results were shown to be in good agreement with the finite element model. This type of analysis could be further extended in order to analyze the structural behavior of the foundation in a global manner.

Rezumat

Ranforsarea terenului de fundare cu incluziuni rigide reprezintă o metodă eficientă și economică în special în cazul amplasamentelor la care fundarea de suprafață ar conduce la tasări inadmisibile în timp ce fundarea de adâncime ar fi mult prea costisitoare. Cu toate că această soluție de ranforsare a stat la baza multor studii de specialitate, există puține referințe asupra efectului incluziunilor la nivelul comportării structurale a fundației. Acest articol a analizat o situație ipotetică în care incluziunile rigide au fost utilizate pentru controlul tasărilor. Accentul s-a pus asupra eforturilor generate la nivelul fundației de prezența incluziunilor și posibilitatea de a considera această influență în proiectarea fundației. Problema a fost studiată într-o primă etapă printr-o abordare de tip celulă unitară, în programul de analiză cu element finit, Plaxis 2D. Valorile obținute pentru eforturile în fundație au fost verificate apoi printr-o abordare analitică. În final, problema a fost analizată cu ajutorul unui model de tip Winkler cu coeficienți elastici variabili, în programul GEO5 – Plate. După o calibrare a parametrilor elastici, s-a observat că

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rezultatele obținute sunt în acord cu cele din modelul cu elemente finite. Acest tip de analiză ar putea fi extins pentru analiza globală a comportării structurale a unei fundații amplasată pe mediu ranforsat.

Keywords: soil reinforcement, rigid inclusion, numerical model, finite element analysis, Plaxis 2D

1. Introduction

The concept of soil reinforcement has been around since ancient times when it was used in the form of timber piles for the foundation of bridge piers [1]. Nowadays this approach is mostly related to the use of granular columns, also known as flexible inclusions, or columns made from bonded materials, known as rigid inclusions. Although it was concluded that the influence of the flexible inclusions on the foundation stresses should be taken into consideration in the design stage of the foundation [2-5], many soil improvement projects neglect this effect as it can be difficult and time consuming in order to evaluate. Thus, the design of soil reinforcement projects generally relies on deformability and/or bearing capacity verifications.

Rigid inclusion reinforcement is a more recent technique of soil improvement. The rigid inclusion material can vary from wood to metal but the most commonly used types are the vibro-concrete, jet grouting, soil mixing and controlled modulus columns. This method also implies the existence of a load transfer layer, made from granular or stabilized soils, which introduces a discontinuity between the foundation and inclusions. Given the high rigidity of the inclusions compared to the surrounding soil, the majority of the load is redirected through the transfer platform towards the inclusions, consequently reducing the fraction of the load transferred to the soil [6]. Although the presence of the load transfer layer significantly reduces the influence that the inclusions have on the foundation bending moments and shear forces, certain situations may require this influence to be considered in the structural design of the foundation. A complex set of guidelines regarding the design, construction and control of rigid inclusion projects is given by the ASIRI recommendations (2012) [7]. In terms of soil - structure interaction, these recommendations present two different methods in order to analyze the behavior of slab-on-grades supported by soil reinforced with rigid inclusions, one of which is the method of differentiated subgrade reaction coefficients.

The present article studied a typical situation of a foundation resting on a rigid inclusion reinforced soil. The aim was to highlight the effect that the presence of the rigid inclusions generates on the foundation behavior. The problem is detailed in Chapter 2 and analyzed in the form of a axisymmetric unit cell model in the finite element program, Plaxis 2D. The finite element model and results are presented in Chapter 3. The resulting values for the foundation bending moments are verified through an analytical calculation in Chapter 4. The differentiated subgrade reaction coefficients method from the ASIRI recommendations [7] is presented in Chapter 5 and applied for the studied situation. The possibility of expanding this method to a global foundation model is also discussed in this chapter. The resulting conclusions are presented in Chapter 6.

2. Problem description

The analyzed situation is that of a raft foundation which transfers a load of 100 kPa to a rather compressible soil. The load could be attributed for example to a five story building. Because placing the raft on the unimproved soil would lead to large settlements, a soil improvement method is taken into consideration by means of vibro-concrete rigid inclusions. While the presence of the rigid inclusions also increases the bearing capacity of the reinforced medium, the present study focused on the use of rigid inclusions strictly from a serviceability limit state requirement. The inclusions are considered to be 10 m long with a diameter of 0.6 m. They are placed in a 3 x 3 m square pattern. A granular layer of 0.5 m thickness is placed on top of the inclusions and the raft foundation is built on top of the granular layer. The soft soil layer is 20 m thick and a practically incompressible layer is considered below it. The problem is illustrated schematically in Fig. 1. It should be noted that the geometrical configuration and dimensions of the reinforcement solution were adopted in the limits indicated by the ASIRI recommendations [7].

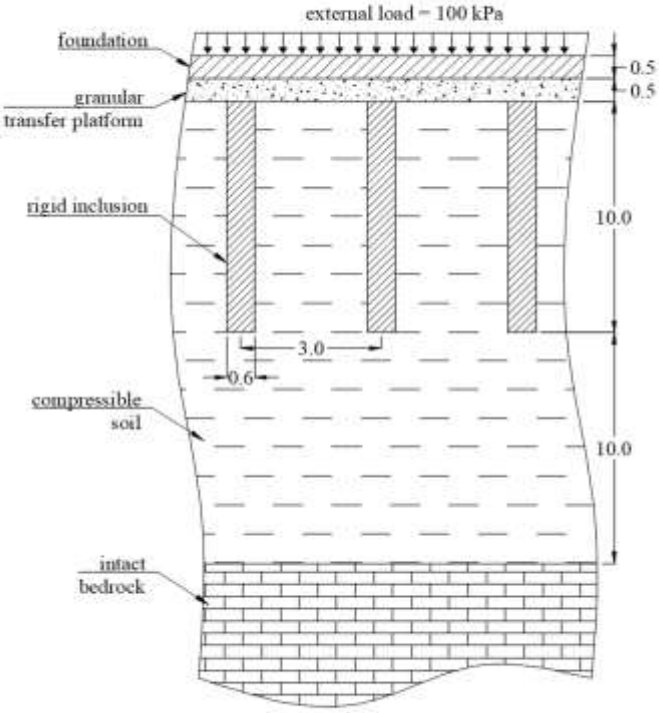


Figure 1. Schematic representation of the analyzed situation (not scaled)

3. Numerical analysis by finite element method

The problem is idealized through the use of an axisymmetric unit-cell approach in the finite element analysis software, Plaxis 2D. This type of model is generally used for the analysis of raft foundations on reinforced soil as it takes into account all of the components that take part in a soil reinforcement project: foundation, natural soil, inclusions and load transfer layer. However, it is important to note that an axisymmetric unit-cell model can only be used to analyze the central region of the foundation, far from the edges, and only in the case of uniformly distributed loads [7].

3.1 Numerical model description

The analyzed unit cell model is illustrated in Fig. 2. The cell diameter was adopted based on recommendations from Balaam and Booker (1981) [2] (see Fig. 3). The soil was idealized by 15-node triangular elements which provide a fourth order interpolation for displacements. The numerical integration includes twelve Gauss points [8]. A fine global mesh coarseness was chosen and a further refinement of the mesh was made for the soil volumes corresponding to the inclusion and transfer platform. The constitutive models and parameters used in the analysis are given in Table 1. They were selected based on recommendations found in literature. Thus, a Hardening Soil model with drained behavior was adopted for the soil and transfer platform as it accounts for the pre-failure non-linear behavior of soils [9] and a Linear Elastic model with non-porous behavior was adopted for the rigid inclusion and base layer. The foundation was modeled with a plate element with the parameters indicated in Table 2. The rigid inclusion was modeled with a volumetric element. The elastic modulus for inclusion was determined with Eq. (1) which considers the long-term behavior of the element [7]. All the other parameters required by the adopted constitutive models were kept as default.

$$E = 3700 \cdot f_{ck}^{1/3} \tag{1}$$

The rigid inclusions were considered with a characteristic value of the compressive strength f_{ck} = 16 MPa and the foundation was considered with f_{ck} = 25 MPa. Interfaces were used for modeling soil-structure interaction. The positions of these interfaces and the values for the strength reduction parameter (R_{inter}) are indicated in Fig. 2-a, together with the axisymmetric model used in Plaxis 2D (Fig. 2-b) and the adopted discretization of the model (Fig. 2-c). The strength reduction parameter relates the interface strength (i.e. the rigid inclusion or foundation friction) to the soil strength (cohesion and friction angle) and its value depends on the type of elements that come in contact [8].

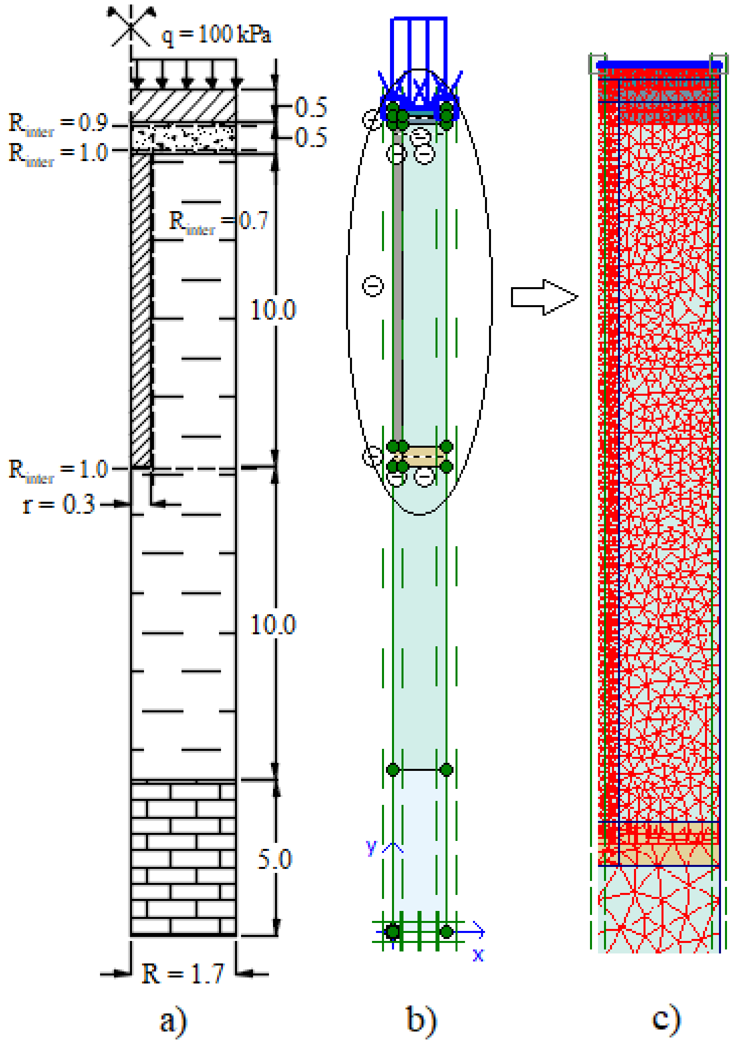


Figure 2. a) schematic representation of the unit cell model (not scaled); b) axisymmetric unit cell model in Plaxis 2D; c) discretization of the unit cell model in Plaxis 2D

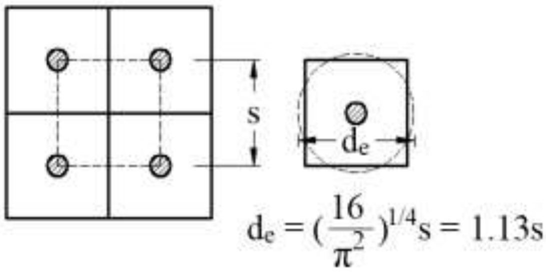


Figure 3. Unit cell diameter for square pattern arrangement of the inclusions [2]

Table 1 Material properties of the soil elements

		Compressible soil	Transfer layer	Rigid inclusion	Base layer
Material model		Hardening Soil	Hardening Soil	Linear Elastic	Linear Elastic
Behavior		Drained	Drained	Non-porous	Non-porous
Parameter	Symbol and Unit				
Unsaturated soil weight	γ_{unsat} (kN/m^3)	18.0	22.0	24.0	20.0
Young's modulus	E_{ref} (MN/m^2)	-	-	9300	3000
Triaxial loading stiffness	E_{50}^{ref} (MN/m^2)	10.0	100.0	-	-
Oedometer loading stiffness	E_{oed}^{ref} (MN/m^2)	10.0	100.0	-	-
Triaxial unloading-reloading stiffness	E_{ur}^{ref} (MN/m^2)	30.0	300.0	-	-
Power for stress-dependent stiffness	m (-)	0.9	0.5	-	-
Poisson's ratio	ν (-)	-	-	0.2	0.2
Cohesion	c_{ref} (kN/m^2)	1.0	0.1	-	-
Friction angle	φ (°)	25.0	35.0	-	-
Dilatancy angle	ψ (°)	0	5.0	-	-

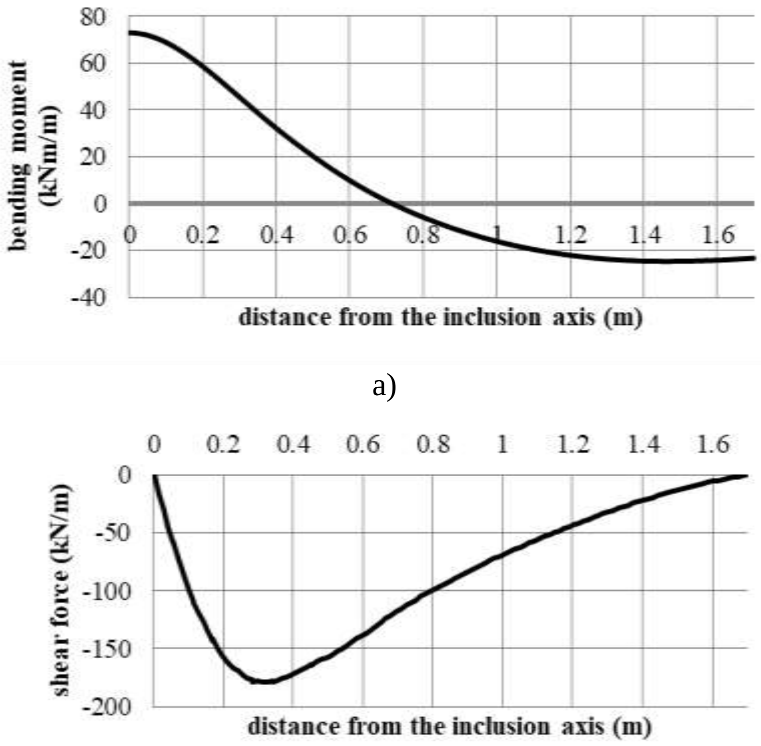
Table 2 Material properties of the plate element

		Foundation
Behavior		Elastic
Parameter	Symbol and Unit	
Normal stiffness	EA (kN/m)	$5.410 \cdot 10^6$
Flexural rigidity	EI (kN/m^2/m)	$1.127 \cdot 10^5$
Equivalent thickness	d (m)	0.5
Weight	w (kN/m/m)	0
Poisson's ratio	ν (-)	0.2

The analysis was carried out in three phases [10]. In a finite element analysis the first phase corresponds to the generation of the initial in situ stresses. This can be done in Plaxis 2D by either the K_0 procedure or by gravity loading. In the present case the K_0 procedure was used as it is recommended in the case of horizontal surfaces [8]. The second phase corresponded to the installation of the inclusions. This was done by activating the inclusion material. The lateral and bottom inclusion interfaces were also activated in this phase. The inclusions were considered *wished in place* (i.e. without considering the effects of inclusion installation on the surrounding soil). The displacements were set to zero after this phase. The third phase comprised of the activation of the remaining components of the model: transfer platform, foundation, distributed load and remaining interfaces.

3.2 Analysis results

An analysis of the unimproved soil model led to a total settlement of 157 mm which is considered to be unacceptable for most constructions. The proposed reinforcement solution led to an efficiency of 64.4% of the system. This value represents the percentage from the total load at the base of the transfer layer (i.e. external load and transfer layer weight) which is transferred to the inclusions. The resulting total settlement in this case was 98 mm (i.e. 37% settlement reduction from the unreinforced case). A further improvement of the total settlement could be achieved primarily by increasing the length of the inclusions and/or by placing them in a denser pattern. In this case however, the obtained settlement reduction was considered to be sufficient.



b)

Figure 4. Bending moment (a) and shear force (b) distribution in the raft foundation

Although the reinforcement of the soil solved the settlement problem, the presence of the rigid inclusions also led to a significant increase in the foundation stresses which generated a maximum bending moment of 73 kNm and a maximum shear force of approximately 180 kN. The variation of the foundation bending moment and shear force within the unit cell is illustrated in Fig. 4. The influence of the inclusions on the foundation stresses could be further reduced by increasing the transfer layer thickness. However, studies have shown that this approach tends to reduce the efficiency of the system thus slightly increasing the total settlement [11]. At the same time, recommendations found in literature state that the transfer layer thickness should not exceed 0.8...1.0 m [7, 10].

4. Analytical analysis of the foundation bending moments

In order to verify the foundation bending moments obtained in the finite element analysis, an analytical calculation was made, based on elastic theory, using a calculation scheme from Bohn (2016) [10] (see Fig. 5-a). As can be seen, this approach requires the establishment of a distribution radius on the base of the foundation ($r_{distrib.}$) of the reaction found at the top of the rigid inclusion. This value was estimated to be close to 0.45 m based on the vertical stress distribution on the base of the foundation from the finite element analysis (see Fig. 5-b).

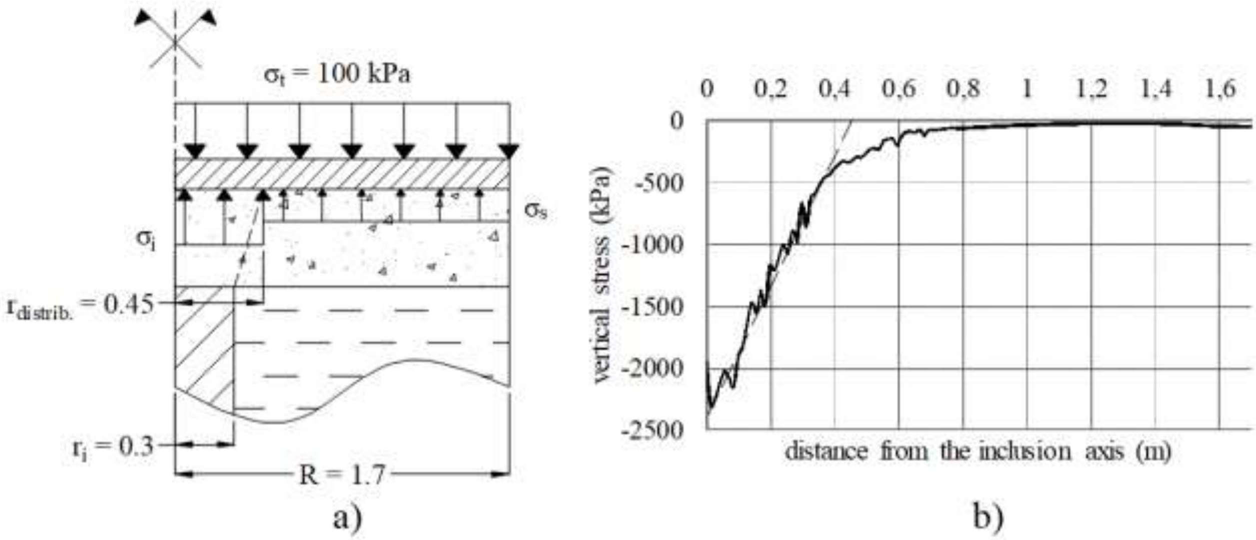


Figure 5. Analytical calculation scheme after Bohn 2016 [10] (a) and vertical stress distribution on the base of the foundation from the finite element analysis (b)

Knowing the inclusion and soil reactions at the base of the transfer layer from the finite element analysis and with the value of the distribution radius, the bending moments at the center (M_c) and edge (M_e) of the unit cell can be calculated using Eq. (2) and Eq. (3) [10]. The value of Poisson's coefficient, ν, for the analytical calculation was taken 0.2. In the absence of a finite element analysis software, analytical models proposed in ASIRI (2012) [7] can be used to estimate the maximum load applied to the inclusions.

$$M_c = \frac{-(r_{distrib.} - \sigma_s)\cdot R^2}{16} \cdot \left(\frac{r_{distrib.}}{R}\right)^2 \cdot (1+\nu) \cdot \left(\left(\frac{r_{distrib.}}{R}\right)^2 - 4 \cdot \ln\left(\frac{r_{distrib.}}{R}\right)\right) + \frac{(\sigma_t - \sigma_s)\cdot R^2}{16} \cdot (1+\nu) \quad (2)$$

$$M_e = \frac{-(r_{distrib.} - \sigma_s)\cdot R^2}{8} \cdot \left(\frac{r_{distrib.}}{R}\right)^2 \cdot \left(\left(\frac{r_{distrib.}}{R}\right)^2 - 2\right) - \frac{(\sigma_t - \sigma_s)\cdot R^2}{8} \quad (3)$$

In order to be able to compare the foundation bending moments from the finite element analysis with the analytical solution an additional analysis was made, where the transfer layer was given a unit weight close to zero. This was done because the weight of the transfer layer is not distributed towards the inclusion and compressible soil in the same manner as the external load. The resulting values for the bending moments at the center and edge of the unit cell from the finite element and analytical analyses are given in Table 3.

Table 3 Numerical and analytical results for the foundation bending moments

	M_c (kNm)	M_e (kNm)
Numerical (FEM)	68.21	21.33
Analytical	61.94	21.89

It can be concluded from the results that the analytical approach is in good agreement with the finite element analysis, with approximately 9% difference for the center bending moment and 3% difference for the edge bending moment.

5. Analysis of the foundation on elastic soil

The axisymmetric finite element analysis is a simplified method of studying a soil reinforcement project. However, as it was previously stated, this type of analysis cannot be applied for particular loading situations and cannot offer information on the global behavior of the soil reinforcement system. Such situations can be studied in their entire complexity only through the use of three dimensional finite element models [7], but the software required for this type of analysis is costly to purchase and the models themselves require a lot of time in order to be built and processed.

The ASIRI recommendations [7] present two simplified methods for analyzing the behavior of slabs on grade in rigid inclusion projects. The first one is an envelope method, based on the concept of additional moments and the second one is based on differentiated subgrade reaction coefficients. The latter is discussed and applied here for the previously analyzed situation.

5.1 The method of differentiated subgrade reaction coefficients [7]

The influence of the rigid inclusion is taken into account in this method by using two different elastic coefficients, k_i and k_s (see Fig. 6). The difficulty relies in establishing a distribution for the two elastic coefficients (i.e. determining the r_k parameter). This is done in a calibration phase by adopting different values for r_k until the resulting bending moments come close to those obtained in the axisymmetric finite element model. The values for r_k, k_i and k_s can then be incorporated in a model that encompasses the entire foundation with the purpose of estimating the bending moments and shear forces under different loading patterns.

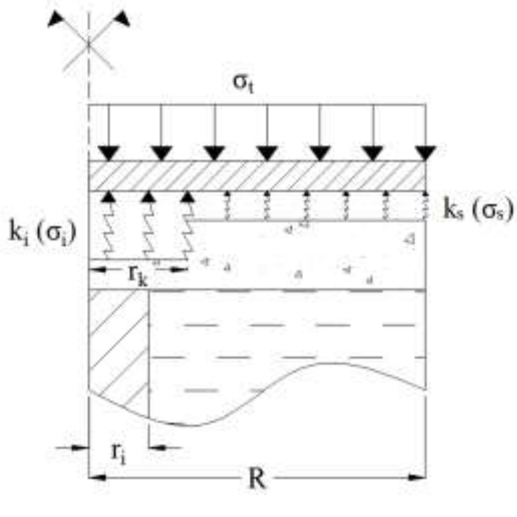


Figure 6. Elastic Winkler model with two reaction coefficients [7]

In the analyzed example the calibration phase was conducted on a unit cell model in the program

Geo5 – Plate. This program uses a Winkler-Pasternak subgrade model, based on C_1 and C_2 parameters, for the design of foundation mats and slabs [12]. The C_1 parameter is actually the elastic coefficient from the Winkler model and the C_2 parameter accounts for shear interaction among the springs. A further simplification was made in the analysis by considering the C_2 parameter equal to zero, thus analyzing the foundation with the classical Winkler soil model. A slight difference in the calibration phase is represented by the shape of the analyzed unit cell. In the finite element model the unit cell has a circular cross section with the radius, R, equal to 1.7 m, whereas in the elastic model the unit cell has a square section with the semi-side, which was kept noted as R, equal to 1.5 m.

The values for k_s and k_i were determined based on the ASIRI recommendations [7]. Consequently, k_s was calculated from Winkler's hypothesis, Eq. (4), considering the minimum reaction directly under the foundation, σ_s, and the maximum settlement, y_{max}, both obtained from the finite element model. The value for k_i was determined based on the σ_i stress, obtained from the equation of conservation of the total load, Q, Eq. (5).

$$k = \frac{\sigma}{y} \tag{4}$$

$$\sigma_i \cdot r_k^2 + \sigma_s \cdot (R^2 - r_k^2) = Q \tag{5}$$

5.2 Results obtained from the Winkler model

Taking into account the distribution of vertical stresses under the foundation from the finite element model, it was assumed that the r_k parameter that would give the best results in terms of bending moments should be between 0.4 m and 0.45 m. These values were thus considered in the calibration phase. The obtained results are presented in Fig. 7, where $M_{sup.}$ is the maximum superior bending moment, found above the inclusion axis, and $M_{inf.}$ is the maximum inferior bending moment. It should be noted that the maximum inferior moment is not found exactly at the edge of the unit cell, but at a slight distance from it.

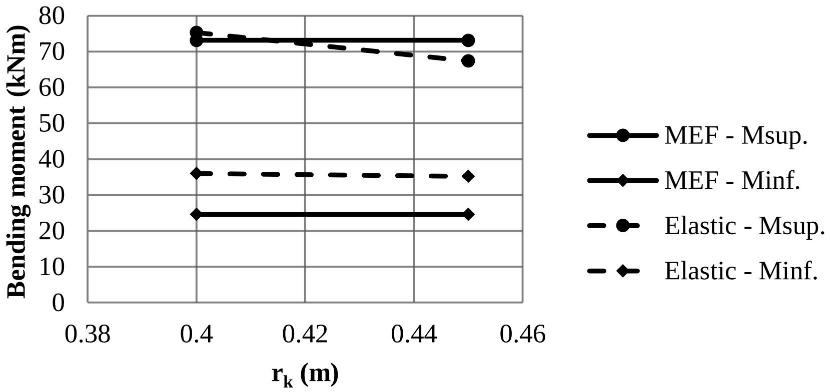


Figure 7. Influence of r_k parameter on foundation bending moments

It can be concluded from Fig. 7 that the best results in terms of bending moments can be obtained for a value r_k = 0.41 m. Table 4 shows the maximum values of the bending moments and shear force obtained in the elastic unit cell model in comparison with those from the axisymmetric finite element model.

Table 4 Comparison of results from the finite element and elastic analyses

Model	R (m)	σ_s (kPa)	σ_i (kPa)	r_k (m)	k_s (MN/m^3)	k_i (MN/m^3)	Maximum settlement (mm)	$M_{sup.}$ (kNm)	$M_{inf.}$ (kNm)	$T_{max.}$ (kN)
reference (FEM)	*1,7*	-	-	-	-	-	*98,61*	*73,11*	*24,63*	*179*
elastic (Winkler)	1,5	17,5	1122	0,41	0,177	11,411	98,46	73,6	35,9	219

Aside from the maximum superior bending moment, which is very close to the value from the finite element model, the maximum inferior bending moment and the maximum shear force have values which are approximately 45% and 22% higher from the ones obtained in the finite element model. However, a structural design of the foundation based on the values from the elastic model would be on the safe side.

Different calibrations are needed for every variation of the magnitude or type of load (distributed and/or point load) on a rigid inclusion mesh. Given that a calibration can only be done with a uniformly distributed load, the resultant of this load needs to be of the same order of magnitude as the resultant of the load for which the calibration is done. Once calibrations have been done for every type of load on the foundation, the patterns can be reproduced as many times as needed and the foundation can be analyzed in a global manner [7].

A known drawback of the Winkler model is the general “dish” settlement profile of the foundation under uniform loads, which does not resemble the real behavior. Even in the case of an unreinforced soil profile, different values of the elastic coefficient should be adopted for the center, corner and edge of the raft [13]. Consequently, specific values for the elastic coefficients should be adopted for the edge of the foundation in order to be able to consider the resulting moments and forces in the structural design [7].

6. Conclusions

The present article analyzed a simple situation of a foundation lying on a rigid inclusion reinforced soil. Following an initial analysis in the finite element program, Plaxis 2D it was observed that the soil reinforcement had a significant positive effect on the total settlement but the presence of the rigid inclusions also generated important stresses in the foundation. The bending moments from the finite element model were verified through the use of an analytical approach which gave similar results. Afterwards, the possibility of estimating the global behavior of the foundation through the combined use of a finite element model and an elastic model with differentiated reaction coefficients was discussed. This type of approach, which is presented in the ASIRI recommendations for the study of slabs on grade, requires calibration of the elastic coefficients for every type and magnitude of foundation loadings. However, given the general dish settlement profile of a foundation on elastic soil, a realistic structural analysis of the foundation would require specific values for the elastic coefficients for the edge of the structure. This type of approach may not be at best agreement with a complex 3D finite element model but, as it was shown in the analyzed example, the structural design would be on the safe side.

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